

FOURIER SERIES AND TRANSFORM LAB

BEFORE YOU BEGIN

PREREQUISITE LABS

- ▶ MATLAB
- ▶ Introduction to Oscilloscope
- ▶ Introduction to Arbitrary/Function Generator

EXPECTED KNOWLEDGE

- ▶ Knowledge of Fourier series representations of periodic signals and how to find them
- ▶ Knowledge of how to calculate the Fourier transform

EQUIPMENT

- ▶ TDS3034B Digital Phosphor Oscilloscope
- ▶ AFG3000 Series Arbitrary/Function Generator

MATERIALS

- ▶ BNC Cable
- ▶ Speaker Control Box
- ▶ Sound Card
- ▶ Audio Cable

OBJECTIVES

The purpose of this lab is to introduce you to the ability of the Tektronix TDS3034B oscilloscopes to estimate the spectrum (Fourier transform magnitude) of a signal and the limitations of this estimate. After completing this lab you should:

- ▶ Know how to use and interpret the Fast Fourier Transform (FFT) function on the oscilloscope.
- ▶ Understand how windowing distorts the spectrum estimated by the FFT function.
- ▶ Be able to recognize and interpret Gibbs phenomenon when observed in practice and on oscilloscopes.

INTRODUCTION

Fourier series and the Fourier transform play a vital role in many areas of engineering such as communications and signal processing. These representations are among the most powerful and most common methods of analyzing and understanding signals. A solid understanding of Fourier series and the Fourier transform is critical to the design of filters and is beneficial in understanding of many natural phenomena.

PRELAB

FOURIER SERIES

Almost all periodic signals can be represented as an infinite sum of sine and cosines. This sum is called a Fourier series representation and is defined by the following equation,

$$f(t) = a_v + \sum_{n=1}^{\infty} a_n \cos n\omega_o t + \sum_{n=1}^{\infty} b_n \sin n\omega_o t$$

where a_v , a_n and b_n are the Fourier series coefficients. These coefficients can be calculated by applying the following equations:

$$a_v = \frac{1}{T} \int_{t_o}^{t_o+T} f(t) dt$$

$$a_n = \frac{2}{T} \int_{t_o}^{t_o+T} f(t) \cos k\omega_o t dt$$

$$b_n = \frac{2}{T} \int_{t_o}^{t_o+T} f(t) \sin k\omega_o t dt$$

Answer Questions 1 – 2.

FOURIER TRANSFORM

A Fourier transform can be used to analyze a circuit in the frequency domain much like the Laplace transform can be used to analyze circuits in the s domain. The Fourier transform is defined by

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

and the inverse Fourier transform is

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} dt.$$

Answer Questions 3 – 5.

WINDOWING

In practice, signals can only be measured over a finite time interval. Mathematically, this can be represented by multiplying the signal by another signal, called a window, that is non-zero for

only a finite interval of time. There are many different types of window functions. In this lab we will analyze rectangular windows like the one shown in Figure 1.

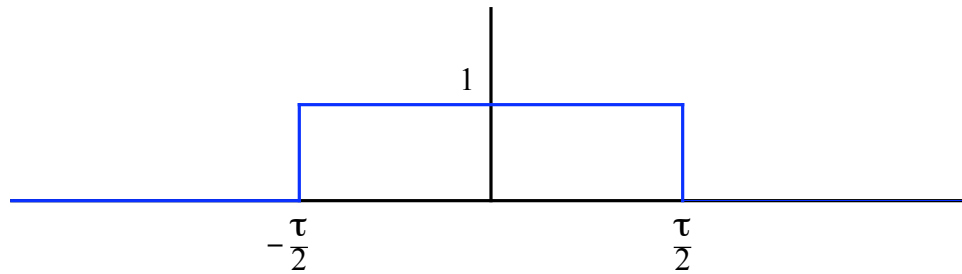


Figure 1. A Rectangular Window.

Answer Questions 6 – 9.

Multiplying the window function with a signal effectively truncates the signal to the duration of the window. Longer windows contain more information about the signal than shorter windows. Consequently, the spectrum of a signal can be more accurately approximated with longer windows. Mathematically, multiplication of a window function with a signal has a spectrum equal to the convolution of the signal spectrum and the window spectrum. If the window is sufficiently long, it will resemble a delta function and the convolution will be very similar to the spectrum of the signal. If the window function is short, the window function will have a spectrum that is bump shaped and the convolution will distort the spectrum of the signal. Visually, the spectrum of the product of the window function and the signal will appear as a blurred or smeared approximation of the signal spectrum.

FOURIER TRANSFORMS ON THE OSCILLOSCOPE

SINUSOIDAL WAVEFORMS GENERATED BY THE FUNCTION GENERATOR

The TDS3034B Oscilloscope can be used to estimate the Fourier transform of a signal. To access this feature on the oscilloscope, press the MATH button and select FFT (Fast Fourier Transform).

Using the Arbitrary/Function Generator, create a sine wave with amplitude of 1 V and a frequency of 1 kHz. Connect the output of the function generator to channel 1 of the oscilloscope and press AUTO SET. Press MATH and then select FFT. You should see a red signal appear on the screen of the oscilloscope. Set the FFT vertical scale to LINEAR and the window to RECTANGULAR.

Remove the menu and the sine wave from the oscilloscope display. Position the FFT estimate until the spike is centered in the screen. Adjust the horizontal scale to its minimum setting (625 Hz). Your screen should be similar to Figure 2.

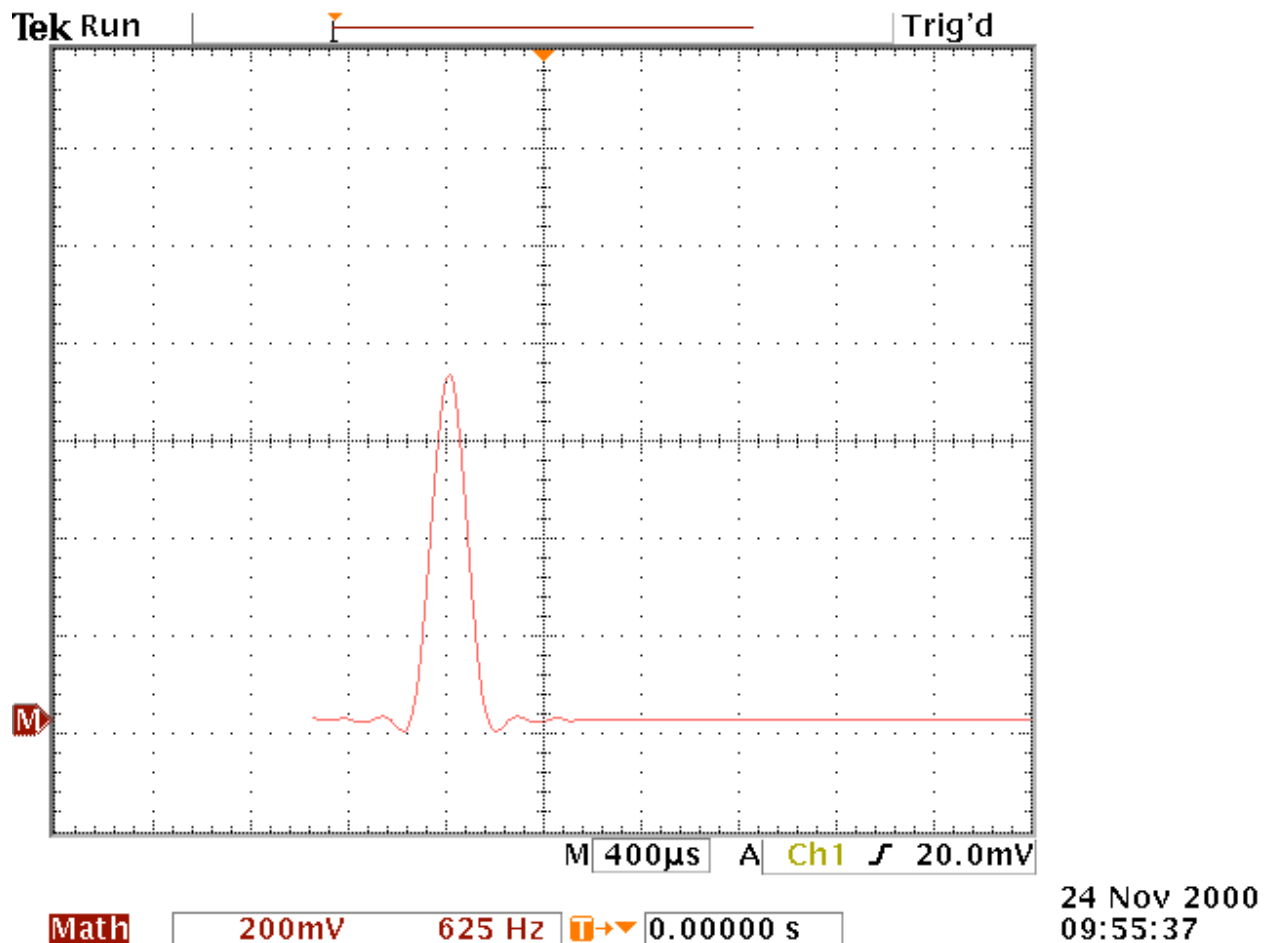


Figure 2. FFT Signal on TDS3034 Oscilloscope.

Answer Questions 10 – 13.

Re-enable the sine wave so it is visible on the oscilloscope display. Set the time division of the sine wave to 1.00 ms. Save the FFT signal to a comma delimited spreadsheet file (“Introduction to Measurement Equipment” lab description). Change the time division of the sine wave to 40 ms and save the FFT signal to a spread sheet file. The following MATLAB example may help you plot your waveforms.

```
x = load('Tek00000.csv');
stem(x(:,1),x(:,2));
xlim([900 1100]);
```

Answer Questions 14 – 15.

OTHER WAVEFORMS GENERATED BY THE ARBITRARY/FUNCTION GENERATOR

Change the signal from the function generator to a square wave with amplitude of 1 V and a frequency of 5 MHz. View the trace of the square wave on the oscilloscope.

Answer Questions 16 – 19.

WAVEFORMS GENERATED BY MATLAB

By now, you should be familiar with generating and listening to different waveforms in MATLAB. In the MATLAB command window, type in the following commands:

```
» fs = 8192;  
» t = (1:500000)/fs;  
» ysine = sin(2*pi*1000*t);
```

fs is the sampling frequency and t is the number of samples. The variable ysine is a vector representing a 1 kHz sine wave.

Answer Questions 20 – 22.