

Experiment Name: Energy_Storage_in_Inductors

Overview



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Fundamental Concepts in Electrical Engineering Lab5 - Transient Currents and Energy Storage in Inductors

Objectives

- Learn an intuitive and an analytical view of inductance and inductors
- Analyze the mechanisms of energy storage in inductors
- Built an experimental circuit to study the transient transfer of energy from an inductor to the circuit

Components

- Resistor: 1 kOhm (10%, 0.25W)
- Inductor: 1mH
- Push-type electric switch (normally open)
- Circuit prototyping breadboard
- Electric wires

Equipment

- Tektronix TBS 1202B-EDU Oscilloscope
- Function Generator (1Hz - 2MHz)
- Power Supply (5V 1A)

Theory

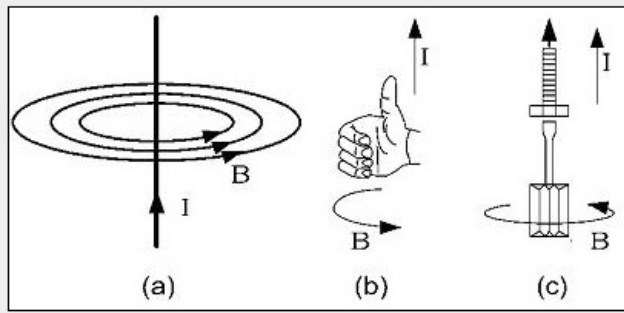
What is Inductance

Most of us are familiar with the inductance of coils and how it relates to geometry, number of turns, and magnetic permeability of the core material. We also learned that Maxwell equations can be used to solve the magnetic fields of any given structure. However, for all other than very basic geometrical objects, the direct application of Maxwell equations becomes very complicated. So, in practice, software tools using numerical methods are used to solve the magnetic fields.

While these tools can provide very precise results, the overall accuracy depends mainly on setting up the boundary conditions. This becomes significantly complicated and requires many approximations, especially in the power distribution circuits on chip, package, and PCB. Thus, the accuracy of the field solver tools simulations depends primarily on the human factor. Therefore, when using field solver tools, it is important to understand the physical principles governing the magnetic fields and induction in power distribution circuits. This helps us analyze the circuits, make better trade-off decisions, and improve the performance of our designs. So in the following sections, I will present an intuitive approach for the analysis of inductance.

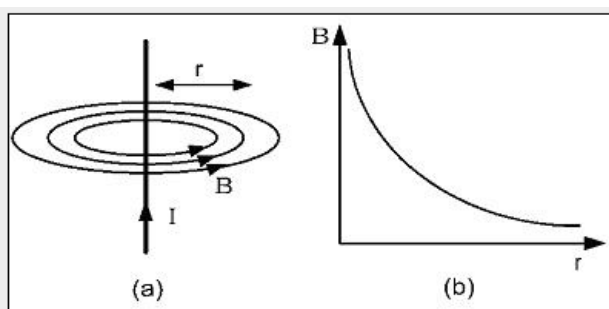
Electric Current Flowing Through a Wire

An electric current flowing through a wire generates circular magnetic field lines around the wire as illustrated in the figure (a) below.



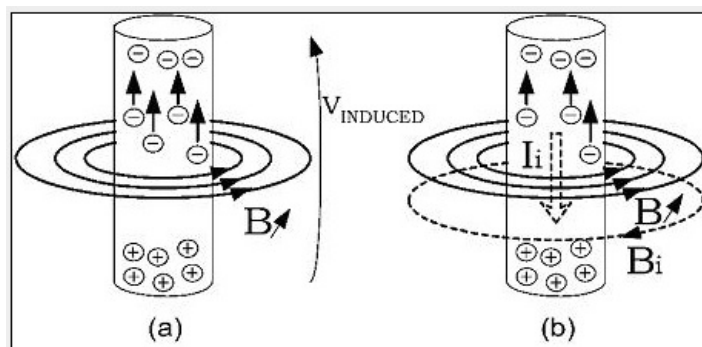
To determine the direction of the magnetic field, we point the right hand thumb towards the current flow and the fingers curl in the positive direction of the magnetic field lines (b). Alternately, we can imagine rotating a screw to advance in the direction of the electric current, and the rotation shows the direction of the magnetic field lines (c).

The magnetic field increases with electric current and decreases rapidly with distance, as shown in the following figure.



Conductive Wire Encircled by Magnetic Field

Let's see now what happens when magnetic field lines surround a wire, as illustrated in the figure below.



If the magnetic field is constant in time, the electric charge inside the wire is not affected. However, if the magnetic field varies, then the electric charge displaces inside the wire inducing a voltage. Figure (a) above illustrates the charge displacement and the induced voltage, $V_{INDUCED}$, generated by an increasing transient magnetic field, B .

To determine the polarity of the induced voltage, we relate the charge displacement to an induced current, i as shown in part (b) of the figure. We then set the direction of this induced current so that its own generated magnetic field, B_i opposes the direction of the initial magnetic field B .

Time Varying Current Flowing through a Wire

We have learned so far that an electric current generates a magnetic field, and a time-varying magnetic field surrounding a conductor induces a voltage in the conductor. We will now combine these two effects to understand what happens when a time-varying current, called here excitation current, flows through a wire.

First, the excitation current generates circular magnetic field lines around the wire, and the intensity of the magnetic field is proportional to the intensity of the current. Consequently, a time-varying excitation current

generates a time-varying magnetic field, which then produces charge displacement and induced voltage back in the same wire.

If we determine the directions of the magnetic field and the induced voltage, we find out that the induced charge displacement opposes the variation of the excitation current. If the excitation current increases, the induced charge displacement current flows in the opposite direction, subtracting from the excitation current. Similarly, if the excitation current decreases, the induced displacement current flows in the same direction, adding to the excitation current.

The “always opposing” charge displacement current can be seen as a reluctance of the wire to changes in the electric current. This reluctance is measured by the quantity called inductance. The inductance uses the symbol L , and it is measured in Henries (H). Analytically, the inductance is defined as the ratio between the magnetic flux and the current generating that flux:

$$L = \frac{\Phi}{I}$$

where

L = inductance (H)

Φ = magnetic flux (Wb)

$\Phi = BA$ for a uniform magnetic field B across the area A

I = electric current (A)

For a single wire the inductance is referred to as self-inductance.

When a variable current flows through a wire, the self-induced voltage into that wire is equal to:

$$V = L \frac{dI}{dt}$$

where

V = self-induced voltage (V)

L = self-inductance of the wire (H)

I = current flowing through the wire (A)

dI/dt = how fast this current varies (A/s)

Self-Inductance of a Straight Wire Segment

Let's start with a very thin (filamentary) wire, which carries an electric current. The magnetic field B is calculated using the equation:

$$B = \mu_0 \mu_r \frac{I}{2\pi \cdot r}$$

where

B = magnetic field (T)

μ_0 = permeability of free space ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m)

μ_r = relative permeability

I = intensity of the electric current (A)

r = distance from the wire (m)

Let's now calculate the self-inductance using as the ratio between magnetic flux and electric current. Knowing the magnetic field from the equation above, we can calculate the total flux by integrating over the area through which the magnetic field lines are passing. The self-inductance will be then equal to the total flux divided by the intensity of the electric current. While this seems relatively easy and straightforward, in reality things are more complicated.

First, the magnetic flux cross-section area expands all the way to infinity, and since the magnetic field depends on materials, we will need to know the magnetic permeability of all the materials surrounding the wire.

Second, for thicker non-filamentary wires the current is distributed within the volume of the wire and generates circular magnetic field lines, both inside and outside the wire. While constant and low frequency currents typically flow uniformly within the wire, high frequency currents concentrate closer to the surface

due to the skin effect. This changes the distribution of the magnetic field inside the wire, and complicates the calculation of the magnetic flux. Notice that due to the skin effect, the magnetic flux inside the wire depends on frequency, and thus the self-inductance depends also on frequency.

As a result, the integration of magnetic flux often becomes impractical, and thus, in the power distribution analysis we often use approximation formulas, which provide reasonably accurate results. We look at self-inductance as a sum of an internal inductance due to the magnetic field lines inside the wire, and an external inductance due to the magnetic field lines outside the wire. The internal inductance of a cylindrical straight wire segment can be calculated as:

$$L_{INT} = \frac{l \cdot \mu_0 \cdot \mu_{r_INT}}{8\pi}$$

where

L_{INT} = internal inductance (H)

l = wire length (m)

μ_0 = permeability of free space ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m)

μ_{r_INT} = relative permeability of the wire material

Notice that the internal inductance does not depend on the wire diameter. This formula is valid only at low frequencies, where the electric current is assumed uniformly distributed inside the wire. At high frequencies, due to the skin effect, the current flows closer to the surface. For very high frequencies, we can assume that all the current flows only on the surface and, thus, no magnetic field exists inside the wire. In this case, the internal inductance becomes zero.

The external inductance can be calculated as:

$$L_{EXT} = \frac{\mu_0 \mu_{r_EXT} l}{2\pi} \left[\ln \left(\frac{2l}{r} \right) - 1 \right]$$

where

L_{EXT} = external inductance (H)

l = wire length (m)

r = wire radius (m)

μ_0 = permeability of free space ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m)

μ_{r_EXT} = relative permeability of the surrounding material

The self-inductance of the wire is the sum of internal and external inductance, $L_{SELF} = L_{INT} + L_{EXT}$. The self-inductance depends on the wire geometry, magnetic permeability of the wire, and magnetic permeability of the surrounding materials. The table below compares the relative magnetic permeability of some materials.

Material	Relative Permeability - μ_r
Copper	1
Aluminum	1
Nickel	600
Iron	5,000
Ferrite	up to 10,000

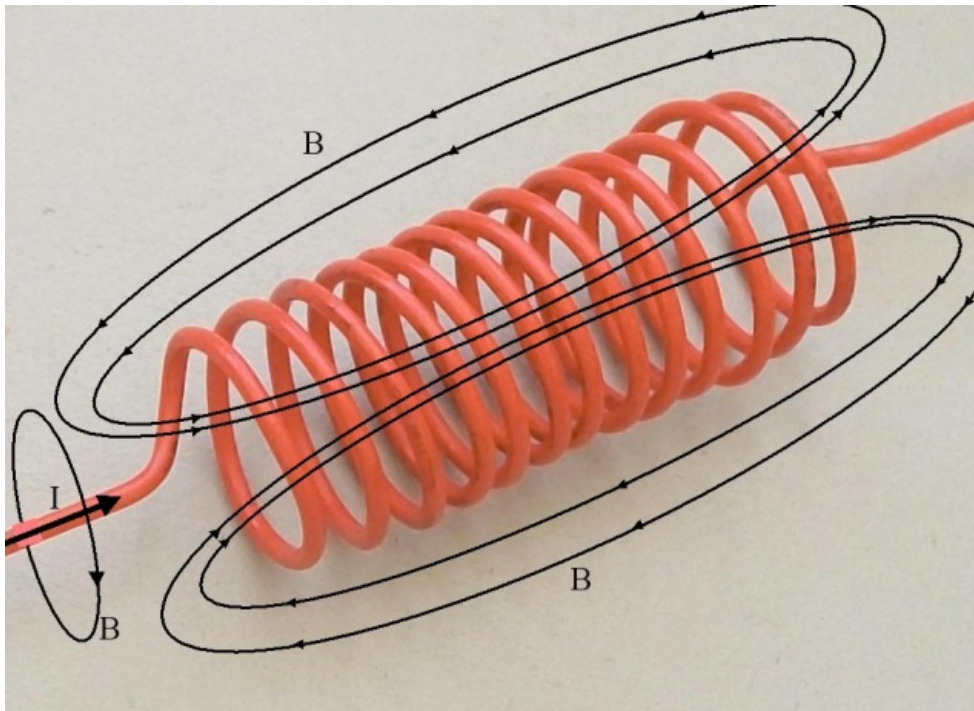
Notice that the inductance can increase by a factor as large as 10,000, depending on the material of the wire and the materials that surround it.

For non-ferromagnetic wires, the self-inductance is dominated by the external component. Thus, in most applications, the self-inductance can be approximated with only the external component, which makes it independent of frequency. We also need to keep in mind that the inductance of a wire increases with length and decreases with thickness. When the wire or surroundings are made of ferromagnetic materials, the inductance increases significantly.

Wanted and Unwanted Inductors

I like to categorize inductors as “wanted” and “unwanted”. I will start first with unwanted inductors: these are the parasitic inductors of various circuits, for example the power distribution to an integrated circuit. Transient currents generated on the silicon chip flow through the power distribution network on the package and printed circuit board (PCB) and due to the parasitic inductance generate transient voltages following the formula $V=LdI/dt$. These transient voltages are seen as power supply noise by the circuits connected to the same power/ground rails. So when designing the power distribution network of an integrated circuit, we want to lower the inductance as much as possible.

In other applications inductors are “wanted”, for example in radio frequency (RF) circuits or switching power supplies, inductors are used as part of the active circuits. In these applications we need much higher inductance than we can obtain from a simple wire. One method is to shape the wire geometry in a coil. The figure below shows the magnetic field generated by electric current flowing through a coil.



Notice: all the way to the left, the circular magnetic field line surrounding the straight wire, as we discussed earlier. As current flows through the coils, these circular magnetic field lines overlap through the center of the coil, creating a high intensity magnetic field inside the coil and increasing the inductance. To further increase the inductance, ferromagnetic materials are placed inside the coil. The inductance of a coil can be calculated using the equation:

$$L = \mu_0 \mu_r N^2 A / \text{length}$$

where:

L=inductance (Henries (H))

μ_0 =permeability of free space (Wb/A*m)

μ_r = relative permeability of the material inside the coil

A= cross-section area inside the coil (m²)

length = length of the coil (m)

Inductors have also parasitic capacitance between coils and series resistance, which makes them RLC resonant circuits at high frequencies.

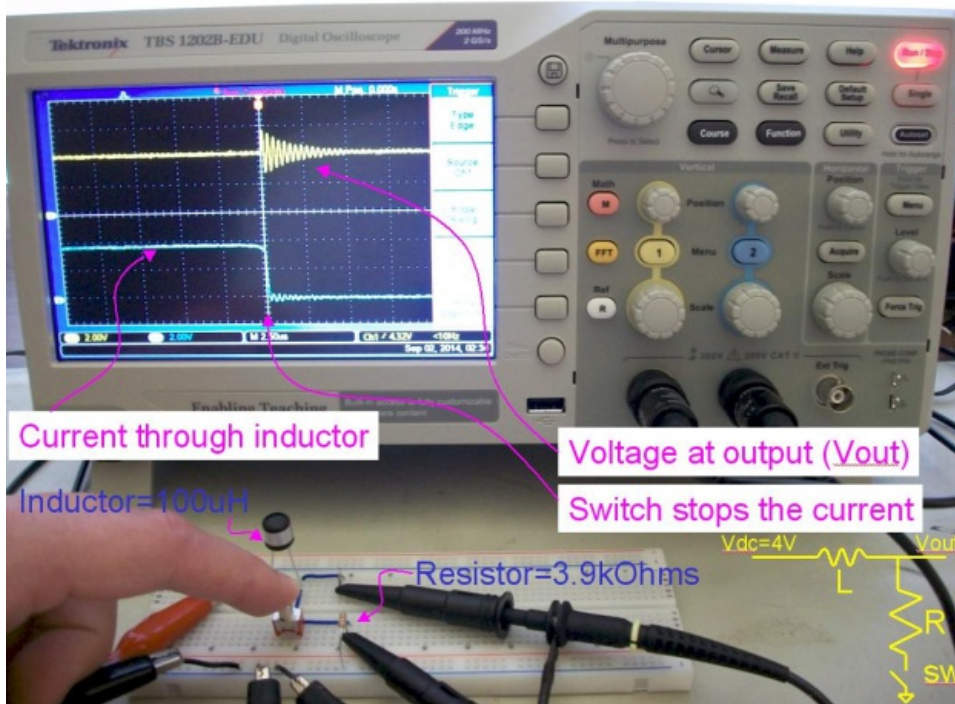
Energy Stored in an Inductor

Inductors store energy when electric current flows through them. The energy stored can be expressed by the formula:

$$W = \frac{1}{2} L I^2$$

Based on this formula, higher inductance stores more energy and higher current flow stores more energy. So what happens with this energy when the current stops flowing? Let's say we have a switch that can interrupt the current flow through the circuit. When we turn the switch OFF, the energy stored in the inductor will maintain the current flow at the output of the inductor until all energy dissipates. However, since the electric circuit is open, this current has no place to go. Intuitively, we can think that electric charge is “pushed” into that node of the circuit and accumulates there. This results in an increase of voltage to very high values, much higher than the nominal voltage supply of typical circuits. As an example, electromagnetic relays have this problem when switching. This is why there is a reverse-biased diode mounted in parallel with the coil. When the current through the coil turns off, the energy stored in the coil dissipates through that diode, avoiding high voltages that would typically damage semiconductor devices.

Let's take a look now at an experiment that stores energy in an inductor and then releases it. The setup is shown in the figure below:



On the breadboard, there is a circuit made of a 100uH inductor in series with a 3.9kOhm resistor and a switch. The schematic is shown, annotated in yellow on the picture. The Vout node is probed by channel 1 (yellow trace) of the Tektronix TBS1202B-EDU oscilloscope. The current through the circuit is probed as voltage drop on the 3.9kOhms resistor using channel 2 (blue trace) of the oscilloscope.

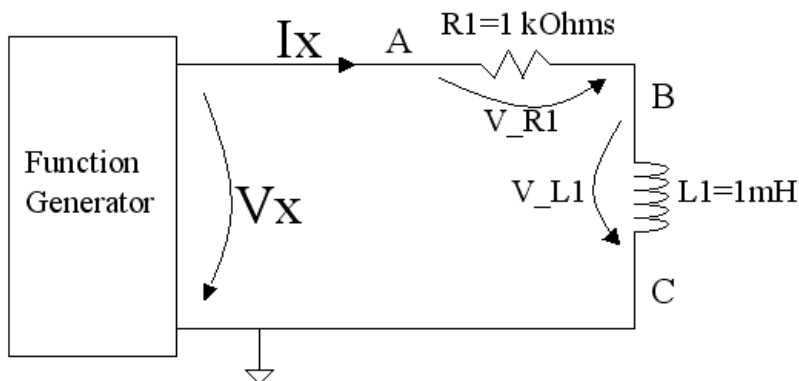
As current flows through the circuit, energy stores in the inductor. Then, when I push the switch button, the flow of electric current is interrupted and the energy stored in the inductor dissipates in the form of electric current pushed into the Vout node. The resulted transient charge accumulation at Vout then dissipates into voltage source node (Vdc=4V node on the schematic) through the resonant RLC equivalent circuit of the inductor, as we can see by the resonant oscillation on the yellow waveform.

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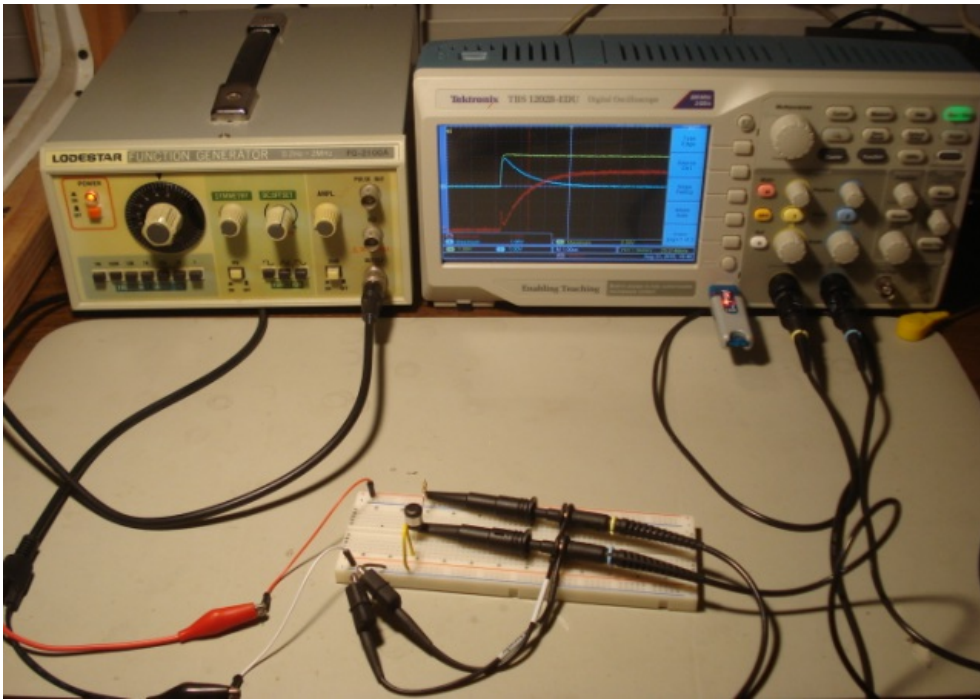
Procedure

Step 1

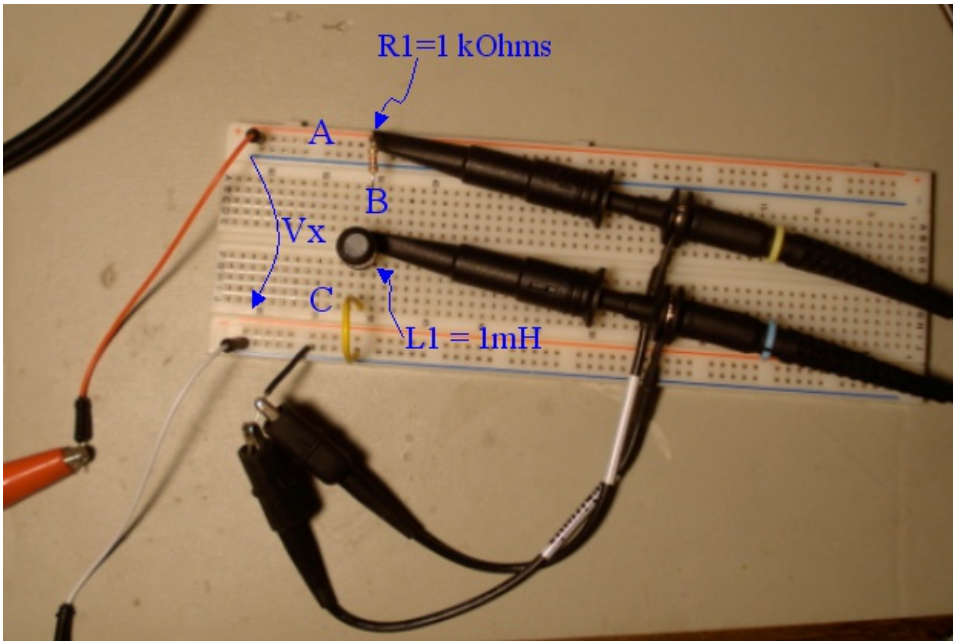
- Construct the circuit showing in the diagram below:



- Here is a picture of the testbench setup:

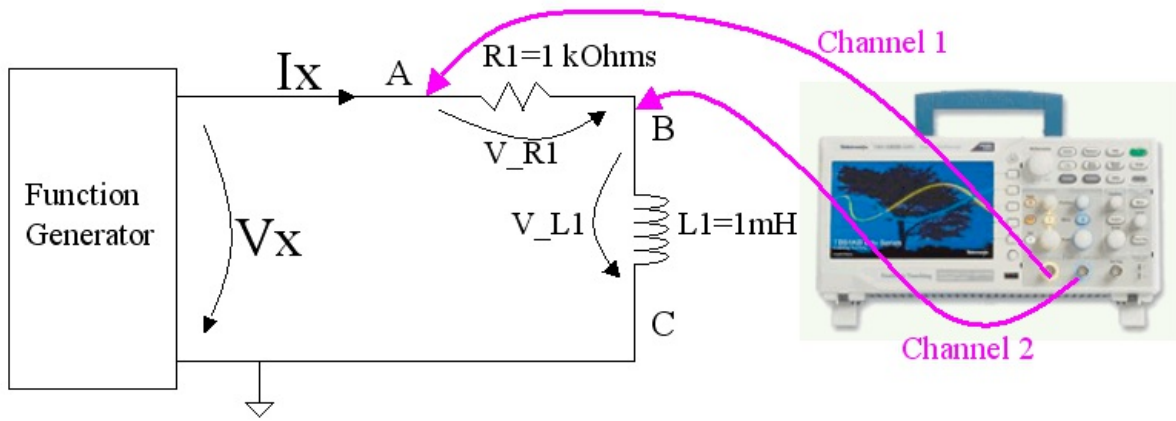


- And here is a more detailed picture of the circuit on the breadboard:

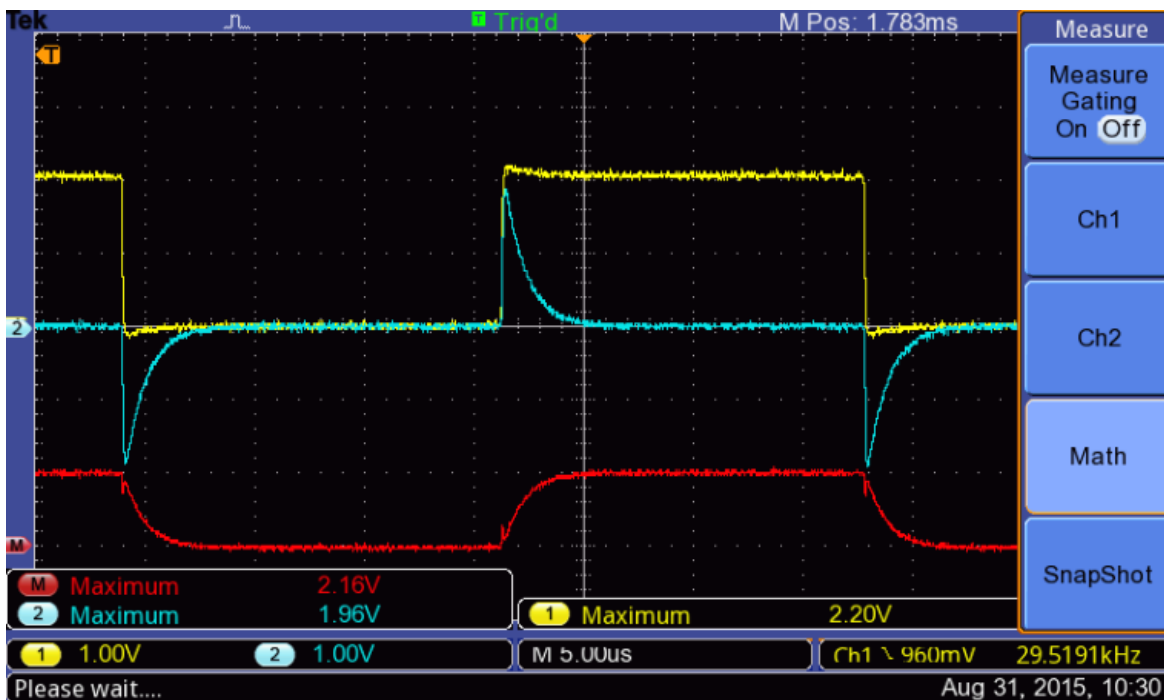


Step 2

- Connect Channel 1 of the oscilloscope to node A of the circuit and Channel 2 to node B, as shown in the diagram below:



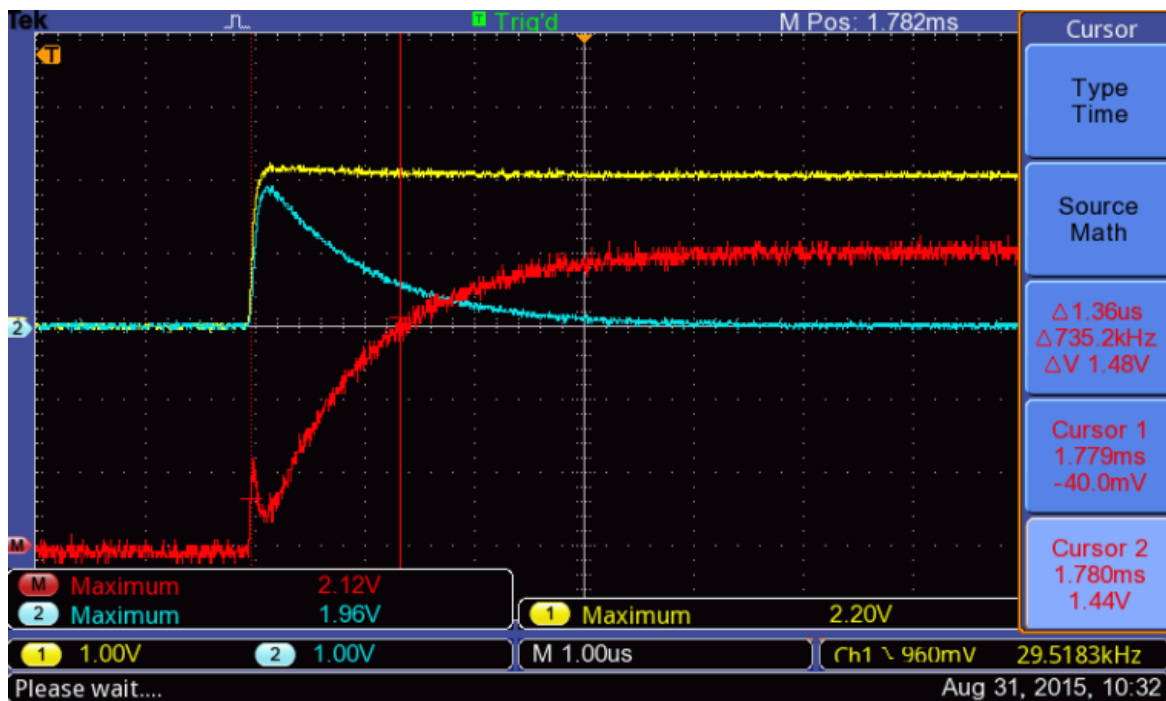
- Set the function generator to rectangular wave mode with amplitude of about 2V and frequency of about 30kHz.
- Set Channel 1 and Channel 2 of the oscilloscope to DC coupling and 1V/div.
- Configure the trigger function to Auto mode, trigger on rising edge, source Channel 1, and adjust the trigger level to be within the range of Channel 1 waveform.
- Configure the built-in Math function of the oscilloscope to display Ch1 - Ch2 with 2V/div.
- The oscilloscope should display similar waveforms as shown in the screenshot below:



- Channel 1 (yellow trace) represents the output signal of the functions generator (node A). When this signal is high (at around 2V level) electric current flows through $R1=1k$ and $L1=1mH$. This current can be determined using Ohm's law from the voltage drop on $R1$, which is measured by the Math trace (red trace).
- Immediately when the voltage at node A increases from zero to 2V there is no current flowing through the inductor and the voltage on the inductor (blue trace) follows Node A voltage.
- Then current starts flowing through the inductor (red trace) and settles to a value close to $V(\text{Node}_A)/R1$, since the resistance of the inductor is very low (we can approximate it to zero). As current starts flowing through the inductor the voltage on the inductor (Node B) starts decreasing towards zero.
- Insert a screenshot of the oscilloscope waveforms in this step.
- Why during discharge the voltage on the inductor becomes momentarily negative?

Step 3

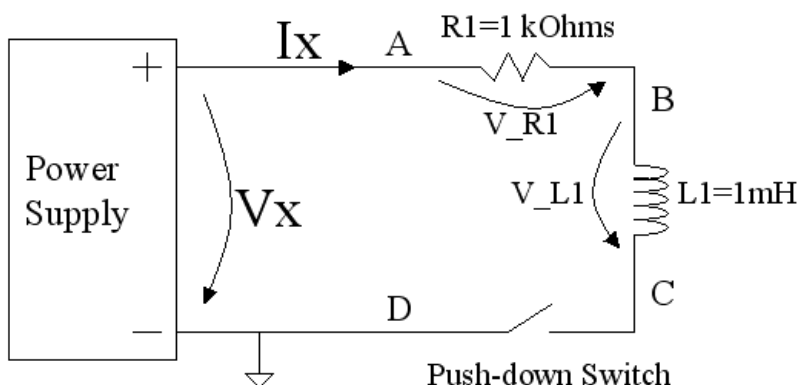
- Use the built-in cursor functions of the oscilloscope to measure the time constant of the current variation through the inductor during the charging phase, as shown in the screenshot below:



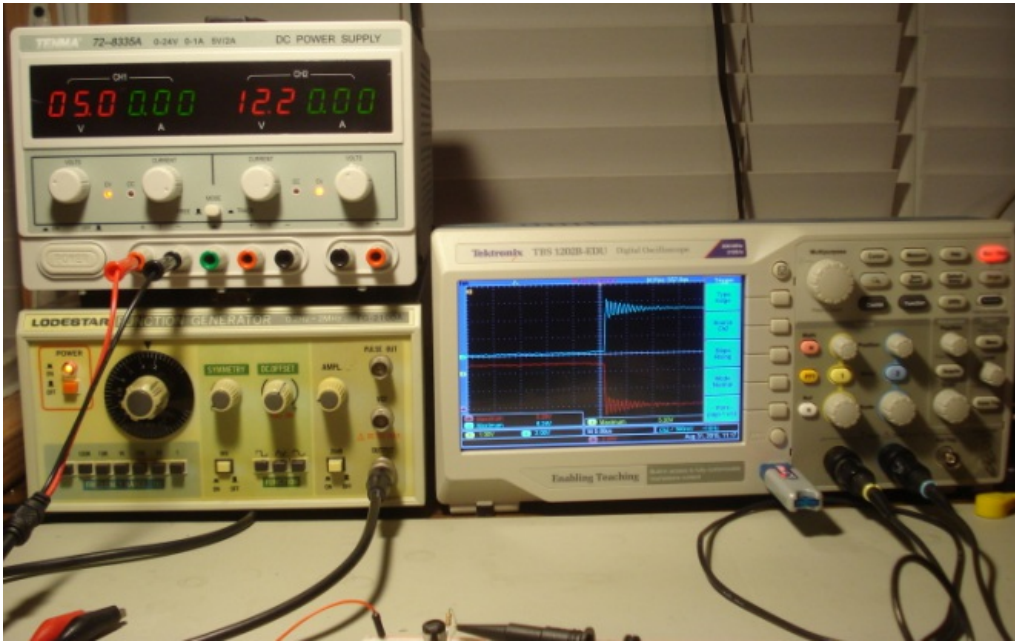
- In the example shown above cursor 1 has been placed at the beginning of the charging phase and cursor 2 has been placed so that $\Delta V = 1.48V$, which is close to $0.707 \cdot 2.12V$ amplitude. The voltage of the Math trace can be translated into current through the inductor using Ohm's law as $I = V_{\text{Math}}(t)/R1$. This With these settings, $\Delta T = 1.36\mu s$ represents the time constant of the circuit, which theoretically should be equal to L/R . If we divide $T=1mH / R=1kOhms = 10\mu s$; however, notice that the measured value is larger = $1.36\mu s$. The difference comes from additional inductance of the wires used in this circuit setup and from the variation of the L1 inductance and R1 resistance from the nominal values.
- Insert a screenshot of the oscilloscope displayed waveforms and cursor readings.
- From the ΔV and ΔT measurements in your experiment determine the measured time constant of the circuit.
- Calculate the time constant and compare the result with the measured value. Comment on any discrepancy between calculated and measured values.

Step 4

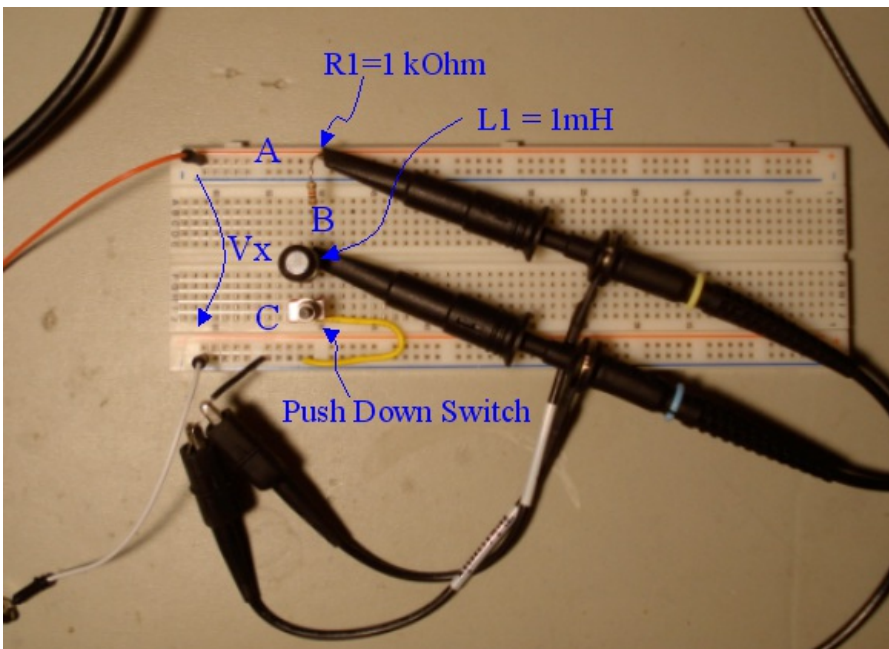
- Construct the circuit showing in the diagram below:



- This circuit is derived from the previous one by adding a switch and replacing the function generator with a power supply. Here is a picture of the testbench setup:

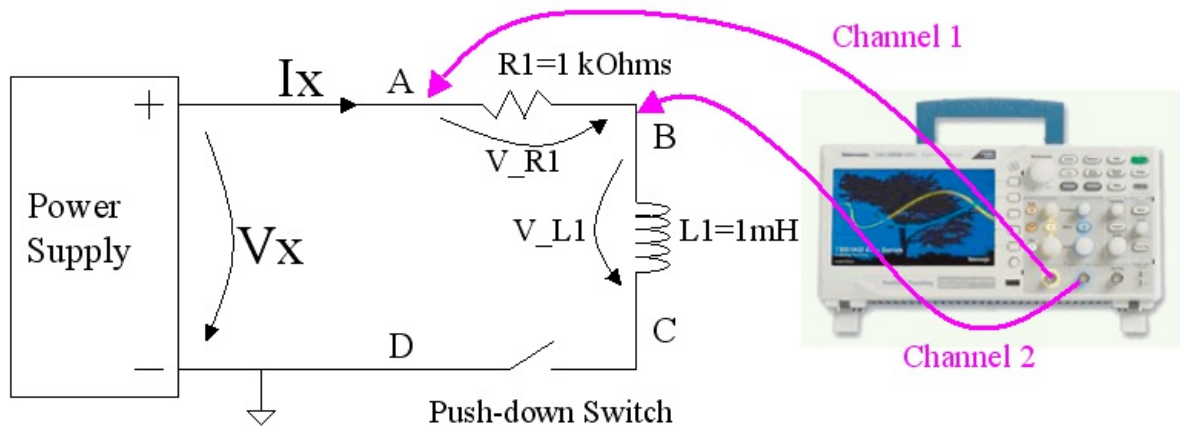


- And here is a more detailed picture of the circuit on the breadboard:

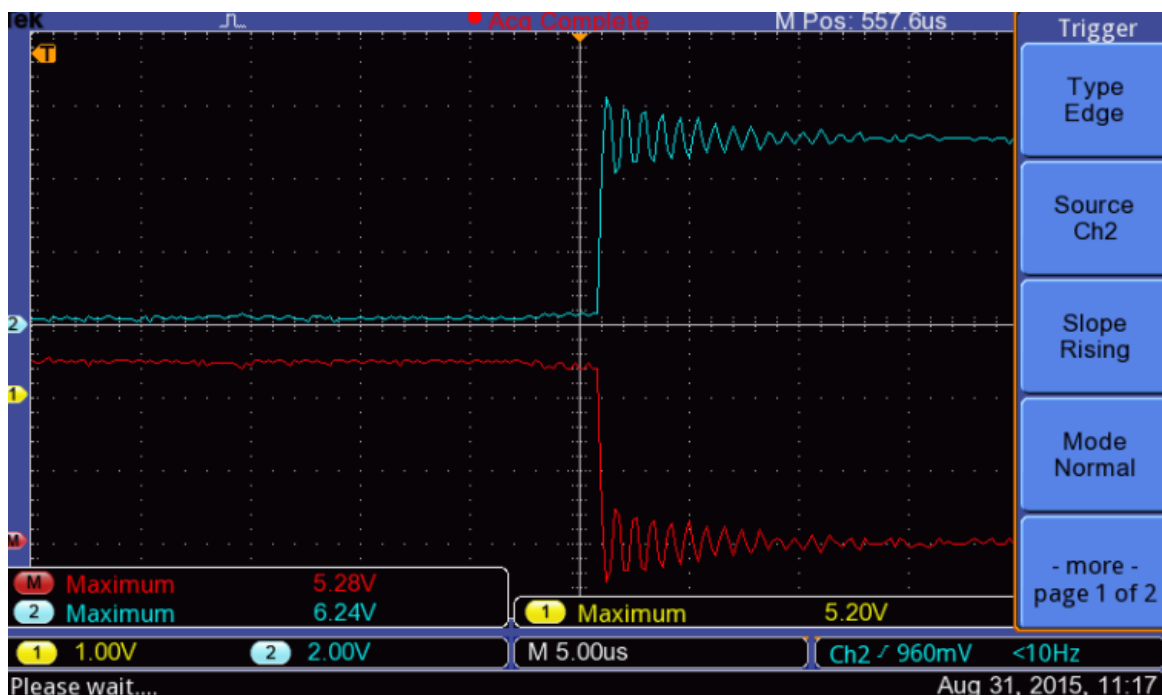


Step 5

- Connect Channel 1 of the oscilloscope to node A of the circuit and Channel 2 to node B, as shown in the diagram below:



- Set the power supply to 5V
- Set Channel 1 of the oscilloscope to DC coupling and 1V/div
- Set Channel 2 of the oscilloscope to DC coupling and 2V/div
- Configure the trigger function to Normal mode, trigger on rising edge, source Channel 2, and adjust the trigger level to about 2V.
- Configure the built-in Math function of the oscilloscope to display Ch1 - Ch2 with 2V/div.
- Set the timebase of the oscilloscope to 25us/div.
- Set the oscilloscope to single acquisition by pressing the "Single" button on the front panel.
- With these settings the oscilloscope will wait for a trigger event to happen and then it will acquire the signals and display them on the screen. The trigger event here is the rising of the voltage on node B after releasing the push-down switch.
- Push down the switch to close the electric circuit and hold it pushed.
- Release the switch. The voltage on node B increases quickly activating the trigger of the oscilloscope which displays a waveform similar to the one in the screenshot below. Notice that the timebase was set to 25us/div; you will need to magnify the timebase to 5us/div after the acquisition completes and the oscilloscope displays the waveforms.



- When the switch is pushed down electric current flows through the circuit resulting in almost zero volts at node B (blue trace in the first half of the screenshot above). The current can be determined using Ohm's law from the voltage drop on R1, which is measured by the Math trace (red trace), in this example $I = 5.28V/1k\Omega = 5.28mA$. The energy stored by this inductor can be calculated as: $W = 1/2 * L * I^2 = 1/2 * 1mH * (5.28mA)^2$.
- Due to this current, energy is stored in the inductor during this time.
- After we release the push-down switch the electric circuit becomes open and current stops flowing. At this moment the energy stored in the inductor is released and it makes the voltage at node B increase above the power supply level of 5V. Due to parasitic capacitance and resistance in the electric path of the circuit and power supply the released energy generates resonance ringing as seen in the second half of the screenshot above. The ringing settles after all the energy stored in the inductor is dissipated, and the voltage at node B settles to 5V.
- Insert a screenshot of the waveforms measured in this step.
- Calculate the current flowing through the inductor during the time the switch is closed.
- From the value of the current calculate the energy stored in the inductor.