OVERVIEW OF NOISE MEASUREMENT METHODS

WHITE PAPER

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Introduction

Noise, or more specifically the voltage and current fluctuations caused by the random motion of charged particles, exists in all electronic systems. An understanding of noise and how it propagates through a system is a particular concern in RF and microwave receivers that must extract information from extremely small signals. Noise added by circuit elements can conceal or obscure low-level signals, adding impairments to voice or video reception, uncertainty to bit detection in digital systems and cause radar errors.
Measuring the noise contributions of circuit elements, in the form of **noise factor** or **noise figure** is an important task for RF and microwave engineers. This paper, along with its associated appendices presents an overview of noise measurement methods, with detailed emphasis on the Y-factor method and its associated measurement uncertainties.

Contents

Appendices

Noise Measurements

The noise contribution from circuit elements is usually defined in terms of noise figure, noise factor or noise temperature. These are terms that quantify the amount of noise that a circuit element adds to a signal. They can be measured directly using available test equipment as well as modeled using both system and circuit simulation SW.

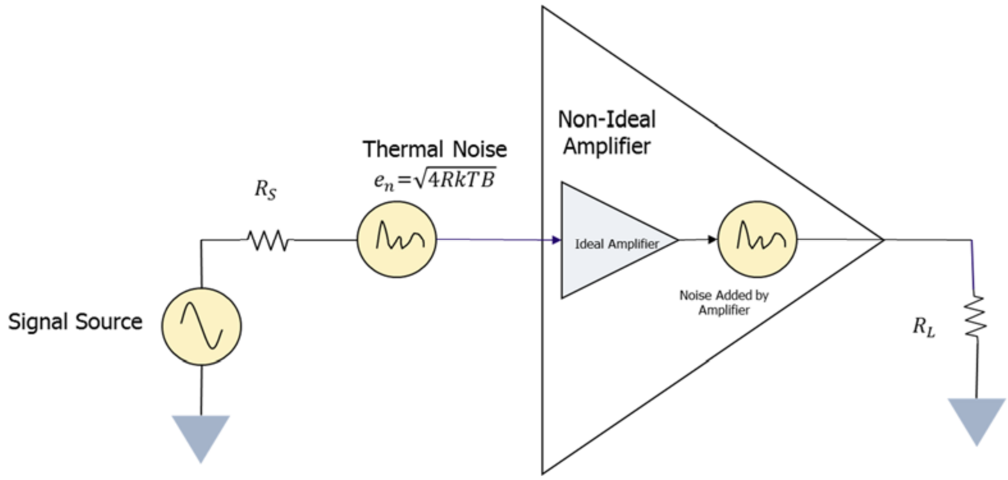

Figure 1. Example of an amplifier with signal, thermal noise and additive noise

Consider the amplifier[1] shown schematically in Figure 1. Its intended job is to amplify the signal presented at its input and deliver it to the load. The thermal noise that is present at the input is amplified along with the input signal. The amplifier also contributes additional noise. The load receives a composite signal made up of the sum of the amplified input signal, the amplified thermal noise and the additional noise contributed by the amplifier. Noise figure, noise factor and noise temperature are figures of merit used to quantify the noise added by a circuit element, the amplifier in this case.

[1] This paper assigns all added noise to the output of the amplifier. Other derivations exist where the added noise is modeled at the DUT input.

Noise Factor, Noise Figure and Noise Temperature

Noise factor is defined as the signal-to-noise ratio at the input divided by the signal-to-noise ratio at the output. Noise factor is always greater than unity as long as the measurement bandwidth is the same for the input and output.

$$\boldsymbol{F} = \frac{(\boldsymbol{SNR})_{\boldsymbol{Input}}}{(\boldsymbol{SNR})_{\boldsymbol{Output}}}$$ **Equation 1**

Noise figure is Noise Factor expressed in dB.

$$\boldsymbol{NF} = \mathbf{10}\ \boldsymbol{log}|\boldsymbol{F}|$$ **Equation 2**

The definitions for noise figure and noise factor are valid for any electrical network, including frequency converting networks that contain mixers and IF amplifiers (up-converters or down-converters).

Active Devices

If we consider an electrical network such as amplifier or frequency converter with input signal $S(\omega)$, voltage gain $A(\omega)$and additive noise referred to the output of $N_A(\omega)$ we have

$$(\boldsymbol{SNR})_{\boldsymbol{Input}} = \frac{|\boldsymbol{S}(\boldsymbol{\omega})|^2}{|\boldsymbol{N}_{\boldsymbol{in}}(\boldsymbol{\omega})|^2},$$

where N_{in} is the noise present at the input to the system. The parenthetical $(\boldsymbol{\omega})$ is used to indicate that these are frequency-dependent quantities. For simplicity, we will drop this functional notation in the remainder of the paper unless it is needed for clarity.

$$(\boldsymbol{SNR})_{\boldsymbol{Input}} = \frac{|\boldsymbol{S}|^2}{|\boldsymbol{N}_{\boldsymbol{in}}|^2}$$ **Equation 3**

An important case exists when the noise at the input is thermal noise, N_T. N_T has a flat power spectral density with power level of $|N_T(\omega)|^2 = kTB$. k refers to Boltzmann's constant, T to the absolute temperature in degrees Kelvin and B to the system bandwidth expressed in Hertz. kTB at 300 Kelvin has a value of $4.14\text{X}10^{-21}$ W or -174 dBm when measured in a one Hz bandwidth.

Similarly, the signal-to noise ratio at the output is given by

$$(\boldsymbol{SNR})_{output} = \frac{|\boldsymbol{SA}|^2}{|\boldsymbol{AN_{in}} + \boldsymbol{N_A}|^2}$$ **Equation 4**

where A is the voltage gain of the device under test (DUT) and N_A is the noise voltage added by the DUT. The noise factor can be computed by taking the ratio.

$$\boldsymbol{F(\omega)} = \frac{\frac{|S|^2}{|N_T|^2}}{\frac{|S\,A|^2}{|A\,N_T+N_A|^2}} = \frac{|A\,N_T+N_A\,|^2}{|A\,|^2|N_T\,|^2}$$ **Equation 5**

It is often more practical to use power gain instead of voltage gain. Let the power gain of the system be

$$\boldsymbol{G} = |\boldsymbol{A}|^2$$

Equation 5 becomes

$$\boldsymbol{F(\omega)} = \frac{\boldsymbol{G}|\boldsymbol{N_T}|^2 + |\boldsymbol{N_A}|^2}{\boldsymbol{G}|\boldsymbol{N_{in}}|^2}$$ **Equation 6**

In the case where the input noise is thermal noise or kT_0B

$$\boldsymbol{F(\omega)} = \frac{\boldsymbol{GkT_0B}+|\boldsymbol{N_A}|^2}{\boldsymbol{GkT_0B}} = \boldsymbol{1} + \frac{|\boldsymbol{N_A}|^2}{\boldsymbol{G\,kT_0B}}$$ **Equation 7**

T_0 in the above equation refers to a standard temperature, usually 290K.

Noise factor and noise figure are an indication of the excess noise (beyond the system thermal noise) contributed by a functional block in a system.

Effective noise temperature refers to the temperature that a matched input resistance would require to exhibit the same added noise.

$|N_A|^2 = G\,kT_e\,B$

$$\boldsymbol{T_e = \frac{|N_A|^2}{G\,kB}}$$ **Equation 8**

Effective noise temperature can be related to noise factor by

$$\boldsymbol{F(\omega) = \frac{G\,kT_0B + |N_A|^2}{G\,kT_0B} = \frac{G\,kT_0B + G\,kT_eB}{G\,kT_0B} = \frac{T_0 + T_e}{T_0} = 1 + \frac{T_e}{T_0}}$$ **Equation 9**

$$\boldsymbol{T_e = T_0[F - 1].}$$ **Equation 10**

T_0 is the reference temperature, usually 290K. Figure 2. Noise Temperature vs. Noise Figure shows a graph of Noise Temperature versus Noise Figure. A noiseless device has a noise temperature of absolute zero or 0 K, while a 4 dB noise figure is equivalent to a noise temperature of approximately 430 K.

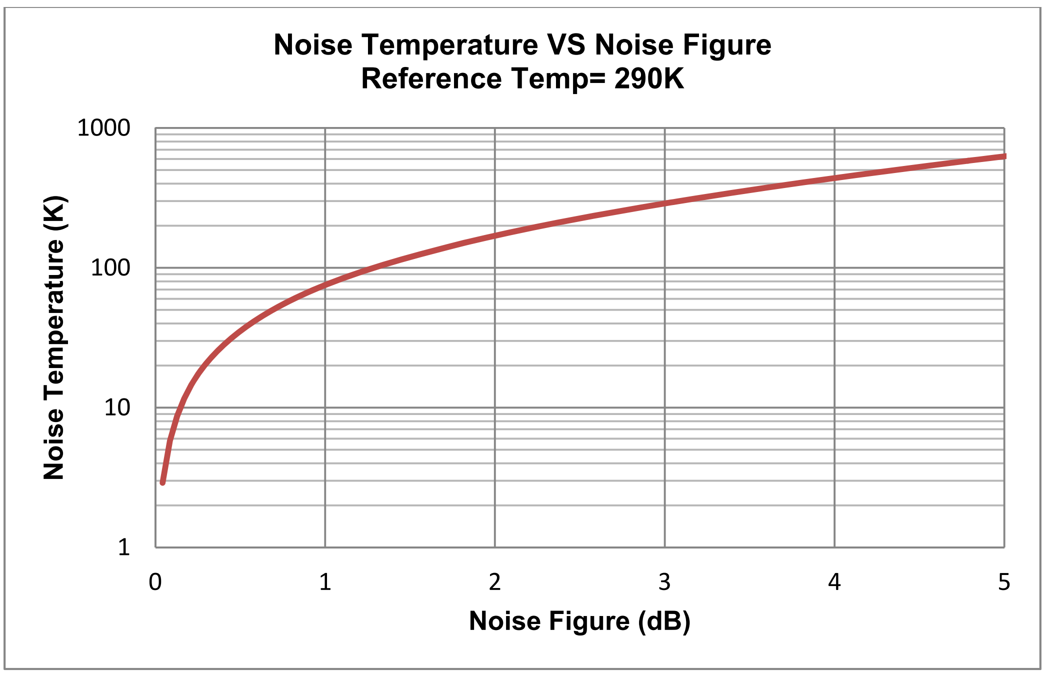

Figure 2. Noise Temperature vs. Noise Figure

Passive Devices

Passive devices, those composed only of resistive or reactive elements, have a power gain less than or equal to unity and contribute no additive noise beyond thermal noise. The noise power at the output when the input is terminated is always kTB. Applying Equation 5 and Equation 6 we have

$$\boldsymbol{F(\omega)} = \frac{(SNR)_{Input}}{(SNR)_{Output}} = \frac{\frac{|S|^2}{\mathrm{kTB}}}{\frac{G\,|S|^2}{\mathrm{kTB}}} = \frac{1}{G} \qquad \textbf{Equation 11}$$

The above equation states that the noise figure of a passive device is the reciprocal of its power gain. A 3 dB attenuator ($\boldsymbol{G = \frac{1}{2}}$) for example would have a 3 dB noise figure.

Noise Figure of Cascaded Stages

Consider a two-port network consisting of two stages. The first stage has thermal noise present at its input. This thermal noise is amplified by the first stage gain and has any additive noise produced by the first stage added to it. The noise at the output of the first stage is then

$$|\boldsymbol{N_1}|^2 = \boldsymbol{G_1\, kT_0B} + |\boldsymbol{N_{A1}}|^2. \qquad \textbf{Equation 12}$$

The second stage has the output of the first stage presented to it. The second stage amplifies the input and contributes additional noise.

$$|N_2|^2 = G_2|N_1|^2 + |N_{A2}|^2 = G_2|N_1|[G_1\, kT_0B + |N_{A1}|^2] + |N_{A2}|^2$$

$$|\boldsymbol{N_2}|^2 = \boldsymbol{G_1G_2kT_0B} + \boldsymbol{G_2}|\boldsymbol{N_{A1}}|^2 + |\boldsymbol{N_{A2}}|^2. \qquad \textbf{Equation 13}$$

The principle illustrated above can be extended for multiple stages.

$$|\boldsymbol{N_k}|^2 = \boldsymbol{G_1G_2 \ldots G_k kT_0B} + \boldsymbol{G_2G_3 \ldots G_k}|\boldsymbol{N_{A1}}|^2 + \cdots + |\boldsymbol{N_{Ak}}|^2 \qquad \textbf{Equation 14}$$

The noise factor is the ratio of the SNR at the input to the SNR at the output. For a given input signal, the ratio for a cascade of k stages is

$$\boldsymbol{F_{ck}} = \frac{(\boldsymbol{SNR})_{\boldsymbol{Input}}}{(\boldsymbol{SNR})_{\boldsymbol{Output}}} = \frac{\frac{|S|^2}{kT_0B}}{\frac{G_1G_2 \ldots G_k|S|^2}{G_1G_2 \ldots G_k kT_0B + G_2G_3 \ldots G_k|N_{A1}|^2 + \cdots + |N_{Ak}|^2}} \qquad \textbf{Equation 15}$$

$$\boldsymbol{F_{ck}} = \frac{(\boldsymbol{SNR})_{\boldsymbol{Input}}}{(\boldsymbol{SNR})_{\boldsymbol{Output}}} = \boldsymbol{1} + \frac{|\boldsymbol{N_{A1}}|^2}{\boldsymbol{G_1kT_0B}} + \frac{|\boldsymbol{N_{A2}}|^2}{\boldsymbol{G_1G_2kT_0B}} + \cdots + \frac{|\boldsymbol{N_{Ak}}|^2}{\boldsymbol{G_1G_2 \ldots G_k}} \qquad \textbf{Equation 16}$$

Applying Equation 6 to Equation 16 yields the noise figure calculation for a system consisting of k cascaded stages. Consider *K* stages in a system. The *k*th stage has power gain G_k and noise factor F_k. Both the signal and the noise from previous stages arrive at the input of the kth stage. The contribution of the kth stage is reduced by the gain preceding it. Noise Figure calculation for the cascade of K stages can be found from

$$\boldsymbol{F_{ck}} = \boldsymbol{F_1} + \frac{\boldsymbol{F_2 - 1}}{\boldsymbol{G_1}} + \frac{\boldsymbol{F_3 - 1}}{\boldsymbol{G_1G_2}} + \frac{\boldsymbol{F_4 - 1}}{\boldsymbol{G_1G_2G_3}} + \cdots + \frac{\boldsymbol{F_k - 1}}{\boldsymbol{G_1G_2G_3 \ldots G_{k-1}}} \quad . \qquad \textbf{Equation 17}$$

$\boldsymbol{Fck} = \boldsymbol{F_1} + \frac{\boldsymbol{F_2 - 1}}{\boldsymbol{G_1}} + \frac{\boldsymbol{F_3 - 1}}{\boldsymbol{G_1G_2}} + \frac{\boldsymbol{F_4 - 1}}{\boldsymbol{G_1G_2G_3}} + \cdots + \frac{\boldsymbol{F_k - 1}}{\boldsymbol{G_1G_2G_3 \ldots G_{k-1}}}$, Equation 17 is often called the Friis formula[2] for cascaded stages. It is named after Danish-American electrical engineer Harald T. Friis.

[2] Friis, H.T., Noise Figures of Radio Receivers, Proc. Of the IRE, July, 1944, pp 419-422.

Effective Noise Temperature of Cascaded Stages

The same equation can be manipulated to give the effective noise temperature for cascaded stages. If we replace the noise factors of each stage by their effective noise temperature we get

$$1+\frac{T_{ck}}{T_0}=1+\frac{T_{e1}}{T_0}+\frac{T_{e2}}{T_0G_1}+\frac{T_{e3}}{T_0G_1G_2}+\frac{T_{e4}}{T_0G_1G_2G_3}+\cdots+\frac{T_{ek}}{T_0\,G_1G_2G_3\ldots G_{k-1}}\,.$$

$$\boldsymbol{T_{eck}=T_{e1}+\frac{T_{e2}}{G_1}+\frac{T_{e3}}{G_1G_2}+\frac{T_{e4}}{G_1G_2G_3}+\cdots+\frac{T_{ek}}{G_1G_2G_3\ldots G_{k-1}}}$$ **Equation 18**

Noise Figure Measurements

Y-factor Method

The Y-factor method uses a noise source that can be switched off and on. It is based on two power measurements, each performed with the same port impedances[3] and the same measurement bandwidth. The Noise source has a specified amount of excess noise. This is specified as the Excess Noise Ratio or ENR. ENR is the ratio of noise from the source to the system thermal noise or kTB, often expressed in dB.

$$\boldsymbol{Y=\frac{P_{on}}{P_{off}}=\frac{kTB\;ENR\;G+|N_A|^2}{kTB\;\;G+|N_A|^2}=\frac{ENR+\frac{|N_A|^2}{kTB\;\;G}}{1+\frac{|N_A|^2}{kTB\;\;G}}}$$ **Equation 19**

$$1+\frac{|N_A|^2}{kTB\;\;G}=F$$

$$Y=\frac{ENR+F-1}{F}$$

$$\boldsymbol{F=\frac{ENR-1}{Y-1}}$$ **Equation 20**

Making a noise figure measurement using the Y-factor method involves the use of a switchable noise source and four power measurements. The first two measurements are used to characterize the noise behavior of the *receiver* used to make measurements. $\boldsymbol{P_1}$, is the power measured by the measuring receiver with the noise source in the *OFF* state. $\boldsymbol{P_2}$, is the power measured by the measuring receiver with the noise source in the *ON* state. The device under test (DUT) is inserted between the noise source and the receiver for the next two power measurements. $\boldsymbol{P_3}$ and $\boldsymbol{P_4}$ are the power measurements made at the DUT output with the noise source turned *OFF* and *ON* respectively.

There are then three steps in making the measurement. The first, often called the calibration step, is to measure the noise figure of the RF receiver used to make the power measurements. The second step is to make a noise figure measurement on the cascaded receiver and DUT. The next step is to de-embed the two measurements.

Let the receiver noise figure be

$$\boldsymbol{F_R(\omega)=\frac{ENR-1}{Y_R-1}}\,.$$ **Equation 21**

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3 Noise sources with a port impedance that changes between the “ON” and “OFF” states contribute additional errors to the noise figure measurement.

The noise figure for the cascade of DUT and receiver has a

$$\boldsymbol{F_c(\omega) = \frac{ENR - 1}{Y_c - 1}} \qquad \textbf{Equation 22}$$

The power gain of the DUT is measured by taking the ratio

$$\frac{P_{onDUT} - P_{offDUT}}{P_{onR} - P_{offR}} = \frac{P_4 - P_3}{P_2 - P_1} = \frac{ENR\ kTB\ G_{DUT} + N_{ADUT} - [kTB\ G_{DUT} + N_{ADUT}]}{ENR\ kTB + N_{AR} - [kTB + N_{AR}]}$$

$$\frac{P_4 - P_3}{P_2 - P_1} = \frac{ENR\ kTB\ G_{DUT} - kTB\ G_{DUT}}{ENR\ kTB - kTB} = G_{DUT} \qquad \textbf{Equation 23}$$

From the cascaded noise figure equation we have

$$F_c = F_{DUT} + \frac{F_R - 1}{G_1} \qquad \textbf{Equation 24}$$

$$F_{DUT} = F_c - \frac{F_R - 1}{G_{DUT}} = \frac{ENR - 1}{Y_c - 1} - \frac{\frac{ENR - 1}{Y_R - 1} - 1}{G_{DUT}}$$

$$F_{DUT} = (ENR - 1)\left[\frac{1}{Y_c - 1} - \frac{1}{G_{DUT}(Y_R - 1)}\right] + \frac{1}{G_{DUT}} \qquad \textbf{Equation 25}$$

Substituting the power ratios for the Y-factors, we get

$$F_{DUT} = (ENR - 1)\left[\frac{1}{\frac{P_4}{P_3} - 1} - \frac{1}{G_{DUT}\left(\frac{P_2}{P_1} - 1\right)}\right] + \frac{P_2 - P_1}{P_4 - P_3}$$

$$F_{DUT} = \frac{(ENR - 1)(P_3 - P_1) + P_2 - P_1}{P_4 - P_3} \qquad \textbf{Equation 26}$$

$F_{DUT} = \frac{(ENR - 1)(P_3 - P_1) + P_2 - P_1}{P_4 - P_3}$, Equation 26 expresses the noise figure of the device under test in terms of the four power measurements in the Y-factor method. This method relying on a series of power measurements is ideally suited to low-level measurement receivers. It has been implemented in modern spectrum analyzers as a cost-effective method of making noise figure measurements.

Cold Source or Network Analyzer Method

The cold source method essentially measures the noise power at the output of a device with an input that is at the reference temperature (cold). It depends on very accurate knowledge of the device gain. Network Analyzers can measure gain with extreme accuracy, making them ideal for this method. Like the Y-factor Method, the cold source method requires a calibration step to determine the measurement receiver's noise figure. This is done with the use of a calibrated noise source and a method similar to what is described in Equation 21.

The gain of the device under test is then measured as a function of frequency using the usual network analyzer methodology. A power measurement is then made as function of frequency with the cold source connected to the device under test. If we let $N_A(\omega)$ be the noise added by the device under test and $N_R(\omega)$ be the noise added by the receiver then the measured power is

$$\boldsymbol{P = kTB\ G + |N_A|^2 + |N_R|^2} \qquad \textbf{Equation 27}$$

$$|N_A|^2 = P - kT_0B\ G - |N_R|^2$$

$$F = 1 + \frac{|N_A|^2}{G\ kT_0B} = 1 + \frac{P - kT_0B\ G - |N_R|^2}{G\ kT_0B}$$

$$\boldsymbol{F = \frac{P - |N_R|^2}{G\ kT_0B}} \qquad \textbf{Equation 28}$$

Some network analyzers[4] offer a noise figure measurement option that includes low noise preamplifiers in their receivers, calibrated noise sources and the software to make measurements. The Network Analyzer's ability to make accurate transmission and reflection measurements means that complete characterization of devices can be made that include Noise Figure and S-parameters, making Network Analyzer measurements ideal for inclusion in software-based system models.

[4] Agilent Application Note: High Accuracy Noise Figure Measurements Using the PNA-X Series Network Analyzer.

Signal Generator (Twice Power) Method

Measuring devices with a high noise figure presents a problem for the popular Y-factor method. The Y-factor approaches unity as the noise figure approaches the source ENR. This affects the accuracy of the Y-factor measurement. The twice-power method uses a signal generator and a measuring receiver with an accurately known noise BW such as a spectrum analyzer. The input to the device under test is terminated with a load at approximately the reference temperature (usually 290K). A signal generator is then connected to the device under test until the measured power is exactly 3 dB or twice the power measured with the input terminated. At this point the sinusoidal power is exactly the same as the noise power and the noise factor can be calculated. Knowledge of receiver bandwidth is critical but of knowledge of device gain is not needed. The noise factor of the cascaded DUT and receiver can be computed from

$$F_c = \frac{P_{Gen}}{kT_0B} \qquad \textbf{Equation 29}$$

The noise factor for the DUT can be dis-embedded using the formula for cascaded noise figure in Equation 24.

Direct Noise Measurement Method

Devices with high noise figure can be measured with directly with a spectrum analyzer or other receivers with accurately known bandwidths as long as the gain is known. The input to device under test is terminated in a source that is near the reference temperature (290K). The noise power at its output is measured and noise factor can be computed from

$$F_c(\omega) = \frac{P_{out}}{kT_0B\,G} \qquad \textbf{Equation 30}$$

Knowledge of receiver bandwidth is required, as is knowledge of device gain. The noise factor of the cascaded DUT and receiver can be computed from the formula for cascaded noise figure in Equation 24.

Noise Figure Measurements in Frequency Converters

The super-heterodyne receiver is at the core of most RF communications systems in use today. The super-heterodyne receiver's ability to provide high gain and frequency selectivity by performing key filtering and amplification functions at a fixed intermediate frequency (IF) makes it the architecture of choice for sensitive receivers ranging from AM radios to the receivers tracking deep space probes.

The core of the super-heterodyne radio receiver is a frequency conversion (mixing), sometimes done in several stages that transfers the information contained spectrum surrounding the carrier to an IF. The noise performance of frequency converters is therefore a key aspect of receiver design.

Measuring the noise figure, noise factor or noise temperature of receivers and frequency converters is similar to the methodology described above for elements that operate at a single frequency, with the exception that the band of frequencies at the input is not the same as the band of frequencies at the output. Care must also be taken to account for the dual sideband nature of mixers and the multiple conversions in samplers and other harmonic mixing devices.

Frequency Converters with Image Rejection

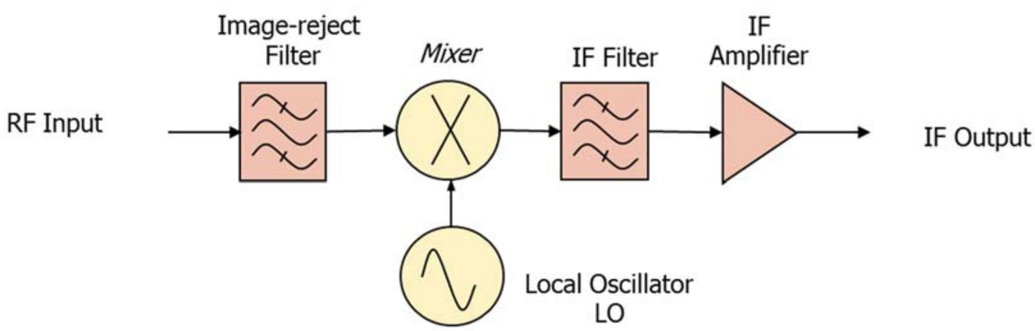


Figure 3. Typical frequency conversion stage.

Typical frequency converters used in RF receivers incorporate an image-reject filter prior to the mixer. The image-reject filter is necessary because of the dual sideband nature of sinusoidal multiplication.

$$\cos(\omega_{RF}\, t)\cos(\omega_{LO}\, t) = \frac{1}{2}cos[(\omega_{RF} + \omega_{LO})t] + \frac{1}{2}cos[(\omega_{RF} - \omega_{LO})t]$$

The sum terms above are rejected by the IF band-pass filter, leaving two possibilities for signals to appear within the IF filter.

$$cos[(\omega_{RF} - \omega_{LO})t] = cos(\omega_{IF}\, t)$$

The above equation has two possible solutions.

$\omega_{RF} = \omega_{LO} + \omega_{IF}$, and

$\omega_{RF} = \omega_{LO} - \omega_{IF}$.

The image-reject filter is designed to pass only the upper or only the lower sideband, but not both as shown in Figure 4.

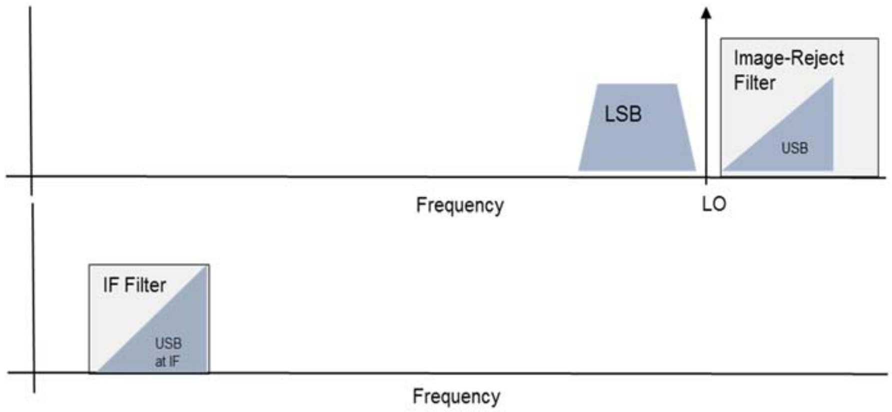


Figure 4. An image-reject Filter selects one of the two possible signal sidebands.

Noise can be represented as a broadband random process with frequency content across all frequencies. Thermal noise is generated in all resistive devices. Shot noise is generated because of the granular nature of electric currents where discrete electrons flow. There are noise generators at every stage in the system shown in Figure 3. Noise components generated prior to the image-reject that lie outside the filter pass-band will be removed by the filter. Both sidebands of noise components generated after the filter will be converted by the mixer. Passive filters are mostly reactive and therefore exhibit no thermal noise. One can then assume that all noise added by the circuit elements in a mixing stage comes from elements located after filtering. Noise from both sidebands is therefore present in the IF, even though the signal from the rejected sideband is not.

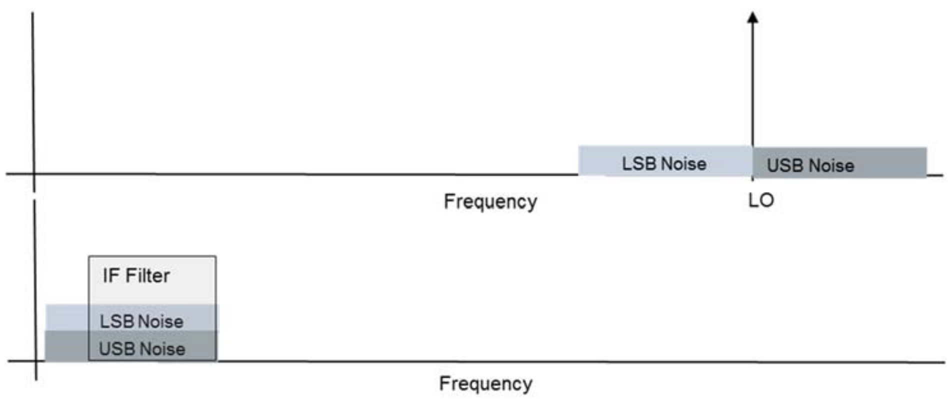


Figure 5. Both upper and lower noise sidebands are converted.

The additional noise from the rejected sideband is indistinguishable from any noise that is added by the system and is generally included in the noise figure measurement of receivers. There are, however, some cases that require special attention.

Image-reject Filter is Included in the Measurement

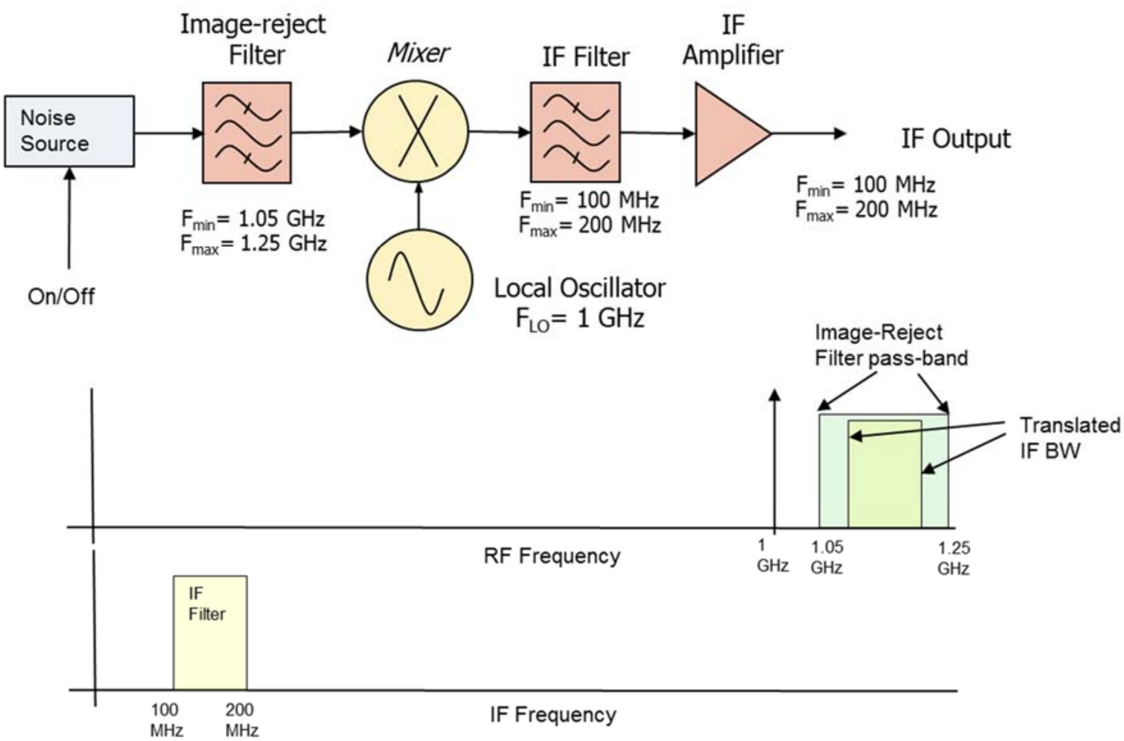


Figure 6. Measuring Noise Figure in a frequency converter that includes an image-reject filter.

Consider a noise figure measurement made in a frequency converter system as shown in Figure 6. The image-reject filter ahead of the mixer allows the only the excess noise from the noise source that falls in the desired sideband (1.05 GHz to 1.25 GHz) to enter the mixer, while thermal noise from both sidebands enters the mixer. Measurements in such a system are straightforward with the exception that the effect of the frequency converter must be considered.

The noise figure measurement is done in two parts as detailed in Equation 20 – Equation 23. The first part measures the Y-factor of the measuring receiver. This is done at the frequency band that exists after the mixer and IF filter. The frequency converter provides a frequency translation so that the Y-factor measurement for the combined DUT and measuring receiver is done at the band of frequencies that exists before the mixer. A spectrum analyzer performing the noise power measurements needs to be tuned so that it covers the 100 MHz to 200 MHz If in its span. The X-axis of a noise-figure VS frequency plot needs to show the equivalent RF frequency, covering 1.1 GHz to 1.2 GHz.

The noise figure measurement for a frequency converter can be examined by modifying Equation 20 – Equation 24 to include the different bands. If we denote frequencies belonging to the band that precedes the mixer as ω_1, and frequencies from the band that exists after the mixer as ω_2, then the noise factor of the receiver, done at the after-mixer band is

$$F_R(\omega_2) = \frac{ENR(\omega_2)}{Y_R(\omega_2)-1}\,. \qquad \textbf{Equation 31}$$

The noise figure for the cascade of DUT and receiver is measured at the mixer-input band is

$$F_c(\omega_{1-2}) = \frac{ENR(\omega_1)}{Y_c(\omega_{1-2})-1} \qquad \textbf{Equation 32}$$

The conversion gain of the DUT expressed as a power ratio is measured by taking the ratio

$$\frac{P_{onDUT}(\omega)-P_{offDUT}(\omega)}{P_{onR}(\omega)-P_{offR}(\omega)} = \frac{ENR(\omega_1)\,kTB\,G_{DUT}(\omega_{1-2})+N_{ADUT}(\omega_2)-[kTB\,G_{DUT}(\omega_{1-2})+N_{ADUT}(\omega_2)]}{ENR(\omega_2)\,kTB+N_{AR}(\omega_2)-[kTB+N_{AR}(\omega_2)]}$$

$$\frac{P_{onDUT}(\omega)-P_{offDUT}(\omega)}{P_{onR}(\omega)-P_{offR}(\omega)} = \frac{ENR(\omega_1)\,kTB\,G_{DUT}(\omega_{1-2})-kTB\,G_{DUT}(\omega_{1-2})}{ENR(\omega_2)\,kTB-kTB} = G_{DUT}(\omega_{1-2}) \qquad \textbf{Equation 33}$$

The subscript 1-2 in Equation 27 is used to denote that the gain in question is the frequency conversion gain or the ratio of the power at $\boldsymbol{\omega_2}$ to the power at $\boldsymbol{\omega_1}$.

From the cascaded noise figure equation we have

$$F_c(\omega_{1-2}) = F_{DUT}(\omega_{1-2}) + \frac{F_R(\omega_2)-1}{G_{Dut}(\omega_{1-2})} \qquad \textbf{Equation 34}$$

$$F_{DUT}(\omega_{1-2}) = F_c(\omega_{1-2}) - \frac{F_R(\omega_2)-1}{G_{DUT}(\omega_{1-2})} = \frac{ENR(\omega_1)}{Y_c(\omega_{1-2})-1} - \frac{\frac{ENR(\omega_2)}{Y_R(\omega_2)-1}-1}{G_{DUT}(\omega_{1-2})}$$

$$F_{DUT}(\omega_{1-2}) = \frac{ENR(\omega_1)}{Y_c(\omega_{1-2})-1} - \frac{ENR(\omega_2)}{G_{DUT}(\omega_{1-2})[Y_R(\omega_2)-1]} + \frac{1}{G_{DUT}(\omega_{1-2})} \qquad \textbf{Equation 35}$$

It is important to note that there are two different values of ENR used in Equation 35. The ENR of the noise source at the IF (mixer output frequency) is used when measuring the receiver alone. The ENR of the noise source at the mixer input frequency is used when the frequency converter is in the measurement.

Image-reject Filter is Excluded in the Measurement

Modular systems often place image-reject filters in separate modules from mixers and IF filters. It is often useful to make noise figure measurements in a frequency converter that is separated from its intended image-reject filter. Measurements in such a system will include noise components from both the upper and lower sidebands as shown in Figure 7. The measurements must be adjusted to account for the differences in the operating environment, which includes the image-reject filter, and the test environment which does not.

Figure 7shows an example of a measurement where the image-reject filter is not included. The noise source contributes noise at both the upper and lower sidebands, both of which are converted to the same band at IF.

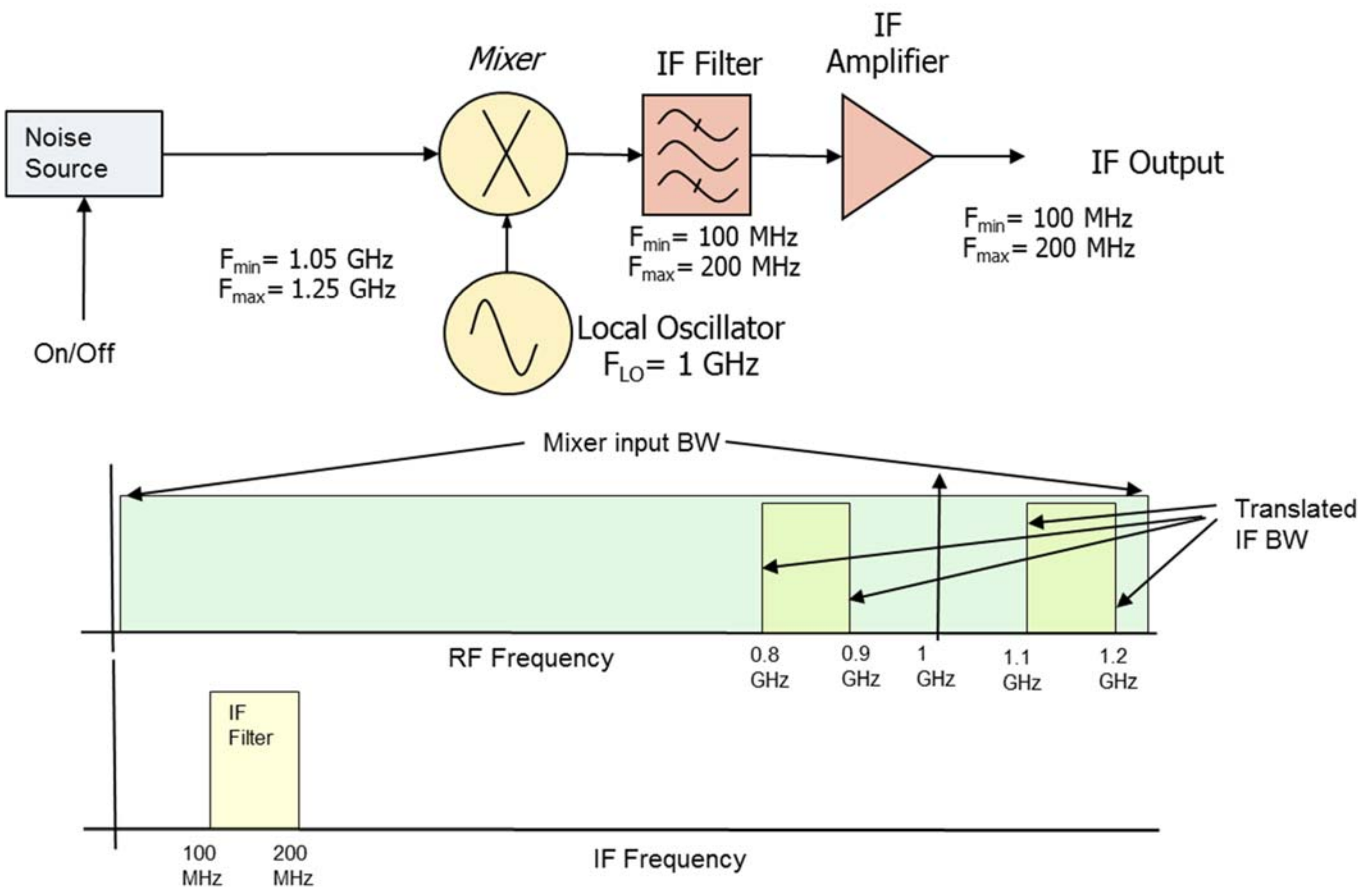


Figure 7. Noise Figure Measurement in a Frequency Converter where the Image-reject Filter is not included in the measurement.

The Y-factor measurement equation for measuring the receiver is unchanged from Equation 18. The Y-factor measurement equation for the double sideband converter must be modified to account for the contribution from both sidebands. If $\mathrm{ENR_U}(\omega_1)$ and $\mathrm{ENR_L}(\omega_1)$ are the excess noise ratio at the upper sideband and lower sideband respectively, $\mathrm{G_U}(\omega_{1-2})$ and $\mathrm{G_L}(\omega_{1-2})$ are the upper and lower sideband conversion gains (or losses) expressed as power ratios and $|\mathrm{N_A}(\omega_2)|^2$ is noise added by the frequency converter stage referred to the post-mixer frequency, then

$$\boldsymbol{Y_{CDSB}(\omega) = \frac{P_{on}(\omega_1)}{P_{off}(\omega_1)}}$$

$$\boldsymbol{Y_{CDSB}(\omega) = \frac{kTB\ ENR_U(\omega_1)\ G_U(\omega_{1-2}) + kTB\ ENR_L(\omega_1)\ G_L(\omega_{1-2}) + |N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}{kTB\ \ G_U(\omega_{1-2}) + kTB\ G_L(\omega_{1-2}) + |N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}}. \qquad \textbf{Equation 36}$$

A double-sideband conversion gain for the frequency converter can be measured by taking the ratio

$$\boldsymbol{G_{DSB}(\omega_{1-2}) = \frac{P_{onDUT}(\omega) - P_{offDUT}(\omega)}{P_{onR}(\omega) - P_{offR}(\omega)}} =$$

$$\frac{ENR_U(\omega_1)\ kTB\ G_U(\omega_{1-2}) + ENR_L(\omega_1)\ kTB\ G_L(\omega_{1-2}) + |N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2 - \{kTB[G_U(\omega_{1-2}) + G_L(\omega_{1-2})] + N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2\}}{ENR(\omega_2)kTB + |N_{AR}(\omega_2)|^2 - [kTB + |N_{AR}(\omega_2)|^2]}$$

$$G_{DSB}(\omega_{1-2}) = \frac{ENR_U(\omega_1)\ \ G_U(\omega_{1-2}) + ENR_L(\omega_1)\ \ G_L(\omega_{1-2}) - [G_U(\omega_{1-2}) +\ G_L(\omega_{1-2})]}{[ENR(\omega_2) - 1]}$$

$$\boldsymbol{G_{DSB}(\omega_{1-2}) = \frac{G_U(\omega_{1-2})[ENR_U(\omega_1) - 1] + G_L(\omega_{1-2})[ENR_L(\omega_1) - 1]}{ENR(\omega_2)\ - 1}} \qquad \textbf{Equation 37}$$

A useful special case of Equation 37 is when the noise source ENR and mixer conversion gain are both constant with frequency.

$ENR(\omega_1) =\ ENR(\omega_2)=\mathrm{ENR}$

$G_L(\omega_{1-2}) = G_U(\omega_{1-2}) = G$

Equation 37 then simplifies to

$$G_{DSB}(\omega_{1-2}) = \frac{G[ENR - 1] + G[ENR - 1]}{ENR - 1} = 2G$$

In this case, the double-sideband gain is twice the gain of the single sideband case.

The Y-factor for the receiver is measured at the band of frequencies that exists after the mixer.

$$F_R(\omega_2) = \frac{ENR(\omega_2)}{Y_R(\omega_2) - 1} \qquad \textbf{Equation 38}$$

The Y-factor of the cascaded frequency converter and receiver can be measured. An analysis of this measurement needs to include both the upper and lower sidebands. Let the subscript U denote the upper sideband ENR and gain and the subscript L denote the lower sideband.

$$Y_{DSB}(\omega) = \frac{P_{on}(\omega)}{P_{off}(\omega)}$$

$$= \frac{kTB\ ENR_U(\omega_1)G_U(\omega_{1-2}) + kTB\ ENR_L(\omega_1)G_L(\omega_{1-2}) + |N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}{kTB\ G_U(\omega_{1-2}) + kTB\ G_L(\omega_{1-2}) + |N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}$$

$$Y_{DSB}(\omega) = \frac{ENR_U(\omega_1)G_U(\omega_{1-2}) + ENR_L(\omega_1)G_L(\omega_{1-2}) + \frac{|N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}{kTB}}{G_U(\omega_{1-2}) + G_L(\omega_{1-2}) + \frac{|N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}{kTB}}$$

From Equation 31,

$$[ENR(\omega_2)\ -\ 1]G_{DSB}(\omega_{1-2}) = G_U(\omega_{1-2})[ENR_U(\omega_1) - 1] + G_L(\omega_{1-2})[ENR_L(\omega_1) - 1]\ .$$

$$Y_{DSB}(\omega) = \frac{P_{on}(\omega)}{P_{off}(\omega)}$$

$$= \frac{kTB\ ENR_U(\omega_1)G_U(\omega_{1-2}) + kTB\ ENR_L(\omega_1)G_L(\omega_{1-2}) + |N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}{kTB\ G_U(\omega_{1-2}) + kTB\ G_L(\omega_{1-2}) + |N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}$$

$$Y_{DSB}(\omega) = \frac{ENR_U(\omega_1)G_U(\omega_{1-2}) + ENR_L(\omega_1)G_L(\omega_{1-2}) + \frac{|N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}{kTB}}{G_U(\omega_{1-2}) + G_L(\omega_{1-2}) + \frac{|N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}{kTB}}$$

In general, noise figure can be computed using Equation 19, repeated here for convenience.

$$F(\omega) = \frac{ENR}{Y(\omega)-1}.$$

We must consider three possible values for ENR when measuring a double-sideband mixer. $\mathrm{ENR_L}(\omega_1)$ and $\mathrm{ENR_U}(\omega_1)$ refer to the excess noise ratio at the lower and upper sidebands at the mixer inputs. $\mathrm{ENR}(\omega_2)$ refers to the excess noise ratio at the band that exists after mixing. Let us define $\mathrm{ENR_{DSB}}\ (\omega_1)$ as an effective double-sideband ENR.

$$\boldsymbol{F_{CDSB}(\omega_{1-2}) = \frac{ENR_{DSB}\ (\omega_1)}{Y_{CDSB}(\omega_{1-2})-1}} \qquad \textbf{Equation 39}$$

$$F_{CDSB}(\omega_{1-2}) = \frac{ENR_{DSB}\ (\omega_1)}{Y_{CDSB}(\omega_{1-2})-1} = \frac{ENR_{DSB}\ (\omega_1)}{\frac{[ENR(\omega_2)\ -1]G_{DSB}(\omega_{1-2})}{\boldsymbol{G_U(\omega_{1-2})}+\ \boldsymbol{G_L(\omega_{1-2})}+\frac{|N_{ADUT}(\omega_2)|^2+|N_{AR}(\omega_2)|^2}{kTB}}}$$

$$F_{CDSB}(\omega_{1-2}) = \frac{[ENR_{DSB}\ (\omega_1)]\left[\boldsymbol{G_U(\omega_{1-2})}+\ \boldsymbol{G_L(\omega_{1-2})}+\frac{|N_{ADUT}(\omega_2)|^2+|N_{AR}(\omega_2)|^2}{kTB}\right]}{[ENR(\omega_2)\ -1]G_{DSB}(\omega_{1-2})}$$

$$F_{CDSB}(\omega_{1-2}) = \frac{[ENR_{DSB}\ (\omega_1)][\boldsymbol{G_U(\omega_{1-2})} +\ \boldsymbol{G_L(\omega_{1-2})}]}{[ENR(\omega_2)\ -1]G_{DSB}(\omega_{1-2})} + \frac{[ENR_{DSB}\ (\omega_1)]\{|N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2\}}{[ENR(\omega_2)\ -1]G_{DSB}(\omega_{1-2})kTB}$$

Let

$$ENR_{DSB}(\omega_1) = \frac{[ENR(\omega_2) - 1]G_{DSB}(\omega_{1-2})}{G_U(\omega_{1-2}) + G_L(\omega_{1-2})} = \frac{G_U(\omega_{1-2})[ENR_U(\omega_1) - 1] + G_L(\omega_{1-2})[ENR_L(\omega_1) - 1]}{G_U(\omega_{1-2}) + G_L(\omega_{1-2})}$$

$$\boldsymbol{F_{CDSB}(\omega_{1-2}) = 1 + \frac{|N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}{[G_U(\omega_{1-2}) + G_L(\omega_{1-2})]kTB}}$$ **Equation 40**

From the equation for cascaded noise factor

$$F_{CDSB}(\omega_{1-2}) = F_{DUTDSB}(\omega_{1-2}) + \frac{F_R(\omega_2) - 1}{G_{DSB}(\omega_{1-2})}$$

$$F_{DUTDSB}(\omega_{1-2}) = F_{CDSB}(\omega_{1-2}) - \frac{F_R(\omega_2) - 1}{G_{DSB}(\omega_{1-2})}$$

$$F_{DUTDSB}(\omega_{1-2}) = 1 + \frac{|N_{ADUT}(\omega_2)|^2 + |N_{AR}(\omega_2)|^2}{[G_U(\omega_{1-2}) + G_L(\omega_{1-2})]kTB} - \frac{\left[\frac{|N_{AR}(\omega_2)|^2}{kTB}\right][ENR(\omega_2) - 1]}{G_U(\omega_{1-2})[ENR_U(\omega_1) - 1] + G_L(\omega_{1-2})[ENR_L(\omega_1) - 1]}$$

$$F_{DUTDSB}(\omega_{1-2}) = 1 + \frac{|N_{ADUT}(\omega_2)|^2}{[G_U(\omega_{1-2}) + G_L(\omega_{1-2})]kTB} + \frac{|N_{AR}(\omega_2)|^2}{kTB}\left[\frac{1}{G_U(\omega_{1-2}) + G_L(\omega_{1-2})} - \frac{1}{G_{DSB}(\omega_{1-2})}\right]$$

For the case where the conversion gains is flat across the upper and lower sidebands $[G_U(\omega_{1-2}) = G_L(\omega_{1-2})]$ and the ENR of the noise source is flat over frequency $[ENR_U(\omega_1) = ENR_L(\omega_1) = ENR(\omega_2)]$, Equation 35 simplifies to

$$\boldsymbol{F_{DUTDSB}(\omega_{1-2}) = 1 + \frac{|N_{ADUT}(\omega_2)|^2}{2\,[G_U(\omega_{1-2})]kTB}}$$ **Equation 41**

It should be noted that the noise factor for the case where the image-reject filter is included in the measurement is

$$F_{DUTU}(\omega_{1-2}) = 1 + \frac{|N_{ADUT}(\omega_2)|^2}{[\boldsymbol{G_U(\omega_{1-2})}]kTB}$$

$$\boldsymbol{F_{DUTDSB}(\omega_{1-2}) = \frac{F_{DUTU}(\omega_{1-2}) + 1}{2}}$$ **Equation 42**

One can extend Equation 42 to the lower sideband. For the case where the upper and lower sideband conversion gains are equal and the ENR is flat.

$$\boldsymbol{F_{DUTU}(\omega_{1-2}) = F_{DUTL}(\omega_{1-2}) = 2F_{DUTDSB}(\omega_{1-2}) - 1.}$$ **Equation 43**

The Nature of Random Noise

Thermal Noise

Everything in the universe is in motion. Even objects that are apparently still have random vibrations of their molecules. This random vibration is felt by the human senses as heat. Temperature is, in fact, a measure of the average kinetic energy of these random vibrations. The same can be said about the molecules in insulators, conductors and semiconductors, charge carriers (electrons and holes) and all physical structures from which electronic devices are built. The power from thermal noise is given by

$$\langle \boldsymbol{P_n} \rangle = \boldsymbol{kTB}. \qquad \textbf{Equation 44}$$

The brackets indicate that this is a statistical quantity expressed as an average. The variables in $\boldsymbol{P_n = kTB}$. Equation 44 are:

P_n = Noise power expressed in watts,

T = Absolute temperature in Kelvin,

B = Bandwidth in Hertz, and

$k = 1.3806488 \times 10^{-23}\ \frac{Joules}{Degree\ K}$ = Boltzmann's constant.

The thermal noise emanating from electronic components manifests itself as fluctuations in voltage and in current. The statistical distributions of voltage and current are nearly Gaussian when observed within a limited bandwidth.

Power Spectral Density of Thermal Noise

Thermal noise in an ideal resistor is approximately white of most of the frequencies used by RF and microwave engineers. A deviation from this flat frequency distribution exists at very high frequencies, when the quantum nature of electromagnetic waves becomes dominant. The noise power can be obtained from[5]

$$\boldsymbol{P_n(f) = \frac{hfB}{e^{\frac{hf}{kT}}-1} + \frac{hfB}{2}}$$ **Equation 45**

Equation 45 includes the photon energy, hf, where h is Planck's constant. In the great majority of cases, the operating frequencies used by RF and microwave engineers are such that $\frac{hf}{kT} \ll 1$ and Equation 46 simplifies to the familiar equation showing a flat power spectral density for thermal noise, $P_n(f) = kTB$. The exceptions occur at very low noise temperatures and very high frequencies.

Figure 8 shows the quantum effects that color the power spectral density of noise. Thermal noise can be considered to have a flat power spectral density out to 100 GHz or more before the quantum effects become dominant.

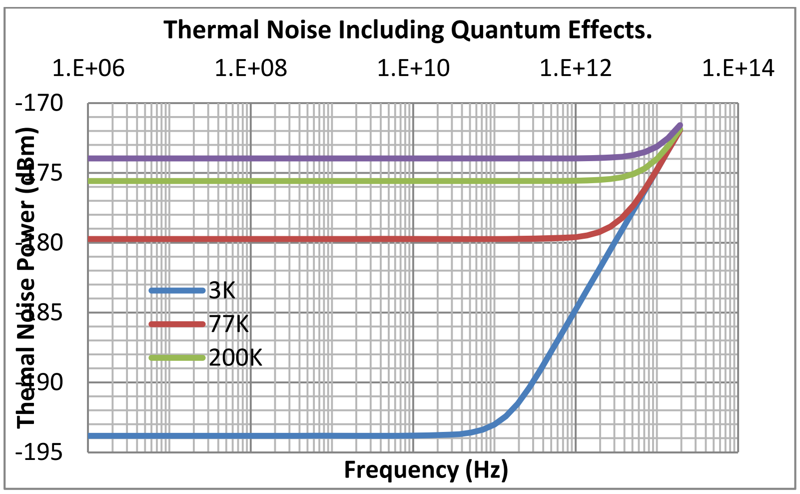


Figure 8. Thermal noise including quantum effects.

[5] Kerr. A.R and Nanda, J. P. 42

Shot Noise

Engineers usually consider electrical current to be a continuous quantity. In reality it is composed of discrete electrons, each with a fixed charge. Current is quantized. Rather than a continuous flow, current is composed of the effects of individual electrons travelling from the source to the load in a circuit. The electrons arrive with a uniform distribution over time. The effect of this variation in time of arrival is called ***shot noise*** or ***corpuscular noise***. Appendix 2 derives the equation for the power spectral density of shot noise. The noise power is proportional to current. The effective noise current approximates a Gaussian distribution with an RMS value or standard deviation given by

$$i_n = \sqrt{2BIq_e}\ .$$

$1/f$ Noise

Random fluctuations with a power spectra density that varies approximately as $1/f$ have been observed in many processes ranging from the flooding patterns of the Nile River to the firing of human brain neurons. Some noise phenomena also exhibit a deviation from white noise at very low frequencies. This so called $1/f$ noise or pink noise has a power spectra density that approximates a curve that inversely proportional to frequency. It can be dominant at low frequencies but drops below the flat thermal noise at frequencies ranging a few Hz to a few kHz, depending on the devices in question.[6]

[6] Johnson, J. B. (1925)

Noise Power Spectral Density Graph

The following graph illustrates the power spectral density of noise from electronic components starting at a frequency of 1 Hz and extending to 10^{14} Hz for at a temperature of 290 K. Low frequencies display the 1/f effects, which are dominant until their power spectral density drops below that of thermal noise. Very high frequencies include quantum effects. The power spectral density can be considered flat for most of the frequencies over which electronic devices operate.

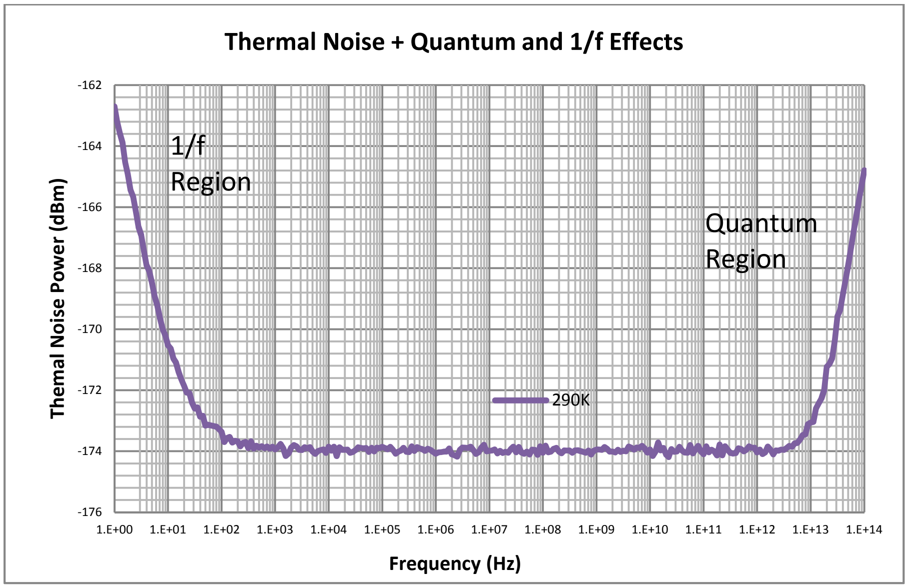


Figure 9. An illustration of noise power spectral density.

Noise in Electronic Components

All physical matter undergoes molecular vibrations as a result of thermal energy. Temperature is in fact a measure of the average kinetic energy of the moving molecules. Electronic devices are no exception. All electronic devices contribute noise at some level due to the vibration of molecules.

Resistors

Resistors contribute thermal noise caused by the random fluctuations of their internal molecules. It is often useful to consider the noise contribution of a resistor in terms of its equivalent noise voltage of its equivalent noise current. These values can be derived from knowledge of kits, the thermal energy of particles, also known as the thermal noise floor. The derivation for the equivalent noise voltage caused by thermal noise in a resistor is shown in Appendix 1. And is given by

$$\langle \boldsymbol{e_n} \rangle = \sqrt{\boldsymbol{4KTBR_0}}$$

The equivalent thermal noise current is given by

$$\langle \boldsymbol{i_n} \rangle = \frac{\boldsymbol{e_n}}{\boldsymbol{R_0}} = \sqrt{\frac{\boldsymbol{4KTB}}{\boldsymbol{R_0}}}$$

The noise power of resistors has a flat power spectral density and depends only on temperature and resistor value.

Capacitors

Ideal capacitors, like all reactive elements do not exhibit thermal noise. Capacitors, however, are made with imperfect conductors and dielectrics and therefore have an associated resistance. The noise voltage from a capacitor can be derived from the noise voltage of a circuit containing a parallel combination of a resistor and a capacitor. Letting the resistor rise to infinity gives us the thermal noise power emanating from the capacitor alone as shown in Appendix 1.

$$enc^2 = \frac{4KTB}{R_p C^2} \frac{1}{\omega^2 + \left(\frac{1}{R_p C}\right)^2} \; .$$

An interesting result occurs when we let the effective parallel resistor go to infinity. The total noise power integrated over all frequencies is only dependent on temperature and on the capacitance value.

$$\int_0^\infty e_{nC}{}^2 \, df = \frac{kT}{C} \; .$$

Inductors
Ideal inductors, like all reactive elements, do not exhibit thermal noise. Real inductive components, however, have the losses in their windings and in their magnetic cores. These losses can be modeled as an equivalent series resistor, which can be show to contribute noise. Appendix 1 shows a derivation of the noise in inductors. It is only dependent on the effective series resistance of the inductor and is given by

$$e_{nL} = \sqrt{4KTB\ \boldsymbol{Re}(\boldsymbol{Z})} = \sqrt{4KTBR_s}\ .$$

Active Devices
Active devices can have many noise contributors. Each of its resistive elements contributes thermal noise. Bias currents contribute shot noise. The internal reactance of all circuit elements shapes the power spectral distribution of noise. It is often useful to model all noise sources in a circuit element in terms of an equivalent noise voltage and current as shown in the Op-Amp in Figure 10.

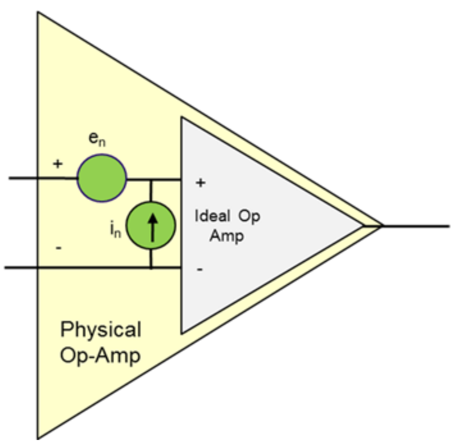


Figure 10. Op-Amp with equivalent Noise voltage and current.

The real op-amp is modeled as an ideal noiseless op-amp with the addition of noise voltages and noise currents at its input. The noise contributions can also be modeled with equivalent noise sources at the amplifier output. The noise voltage and noise current represent the aggregate contributions of all the circuit elements inside the op-amp. Resistive elements contribute thermal. Bias currents contribute shot noise. Reactive elements can shape the noise power spectral distribution. Transistors, RF amplifiers and all active components can be modeled in this fashion. The actual values of the equivalent noise sources can be ascertained with measurements or with careful modeling of the internal circuitry of the amplifier.

Conclusion

Noise exists in all electronic systems. An understanding of noise and how to appropriately measure, model, and account for its effects in a system is an important concern in RF and microwave receivers that must extract information from extremely small signals. Noise added by circuit elements can conceal or obscure low-level signals, adding impairments to the signals being received.

Measuring the noise contributions of circuit elements, in the form of ***noise factor*** or ***noise figure*** is an important tasks for RF and microwave engineers. This paper, along with its associated appendices presents an overview of noise measurement methods, with detailed emphasis on the Y-factor method and its associated measurement uncertainties.

Appendix 1: Noise of a Resistor, Capacitor and Inductor

Resistors

Consider a broadband measurement system with characteristic impedance $Z_0 = R_0 + jX_0$. It has a matched source at its input. We wish to compute the noise contributed by the source impedance.

The source is matched. The source impedance is equal to the conjugate of the system characteristic impedance.

$Z_S = {Z_0}^* = R_0 - jX_0$.

The system noise can be modeled as a noise voltage, e_n. This noise voltage generates a noise voltage in the receiver.

$$e_{meas} = e_n \frac{Z_0}{Z_S + Z_0} = e_n \frac{R_0 + jX_0.}{2R_0}$$

The power received is equal to KTB

$$KTB = \frac{e_{meas}{e_{meas}}^*}{R_0} = \frac{{e_n}^2}{4R_0}$$

The equivalent noise voltage of a resistor is

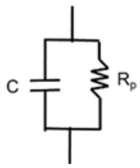
$$\boldsymbol{e_{nR=}\sqrt{4KTBR_0}}$$

The equivalent noise current is

$$\boldsymbol{i_{nR} = \frac{e_n}{R_0} = \sqrt{\frac{4KTB}{R_0}}}$$

Capacitors

The noise voltage for a capacitor can be derived by substituting the impedance for a parallel RC circuit for R in the above equation and then considering resistor values that approach infinity.



A parallel RC circuit has impedance given by

$$Z = \frac{\frac{1}{c}}{j\omega + \frac{1}{R_p C}} = \frac{\frac{1}{c}\left(-j\omega + \frac{1}{R_p C}\right)}{\omega^2 + \left(\frac{1}{R_p C}\right)^2}.$$

We now that noise power is caused by the resistive portion of Z. Substituting the impedance into the equation for noise voltage yields

$$e_{nC=}\sqrt{4KTB\ Re(Z)} = \sqrt{4KTB\frac{\frac{1}{c}\left(\frac{1}{R_p C}\right)}{\omega^2 + \left(\frac{1}{R_p C}\right)^2}}\ .$$

$${e_{nC}}^2 = \frac{4KTB}{R_p C^2}\frac{1}{\omega^2 + \left(\frac{1}{R_p C}\right)^2}$$

Taking the integral over all frequencies and replacing B for Δf, we get

$$\int_0^\infty {e_{nC}}^2\, df = \frac{4kT}{2\,\pi R_p C^2}\int_0^\infty \frac{1}{\omega^2 + \left(\frac{1}{R_p C}\right)^2}\, d\omega\ .$$

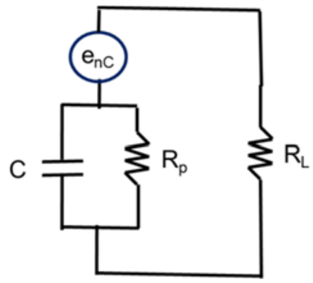
The value of the definite integral is $\frac{\pi}{2} R_p C$.

The equation above gives us

$$\int_0^\infty {e_{nC}}^2\, df = \frac{4kTB}{2\,\pi R_p C^2}\,\frac{\pi}{2} R_p C = \frac{kT}{C}.$$

This is an interesting result in that it tells us that the noise voltage of a capacitor is only dependent on the temperature and the capacitor value.

It is useful to note how much noise power a capacitor will deliver to a resistive load. R_P in the circuit below represents the effective parallel resistance of the capacitor. $R_P \gg R_L$.



If $R_P \gg R_L$, the voltage delivered by the capacitor to a resistive load is

$$e_{nL} = e_{nC}\frac{j\omega}{j\omega + \frac{1}{R_L C}} = \sqrt{\frac{4KTB}{R_p C^2}\frac{1}{\omega^2 + \left(\frac{1}{R_p C}\right)^2}}\quad \frac{j\omega}{j\omega + \frac{1}{R_L C}}$$

The power delivered to that same resistive load is

$$P_n = \frac{\left|e_{nC}\frac{j\omega}{j\omega + \frac{1}{RC}}\right|^2}{R_L} = \frac{4KTB}{R_L R_p C^2}\frac{\omega^2}{\left[\omega^2 + \left(\frac{1}{R_p C}\right)^2\right]\left[\omega^2 + \frac{1}{{R_L}^2 C^2}\right]}$$

If $\gg \frac{1}{R_p C}$, as is the case in the majority of application , then the power delivered to the load is given by

$$P_n = \frac{}{\text{4KTB}}\frac{1}{\left[\omega^2 + \frac{1}{{R_L}^2 C^2}\right]}$$

If $\omega \ll R_L C$ then $P_n = 4KTB\frac{R_L}{R_p}$

The equations above tell us why the thermal noise from capacitors is usually negligible. R_p, the effective parallel resistance of capacitors approaches infinity. The noise power delivered to a load is much smaller than $\boldsymbol{KTB}$.

Piezoelectric Effects in Capacitors

Capacitors can also have additional noise caused by the microphonic effect. [7] Thermal energy and mechanical vibrations both cause random vibrations in the dielectric. These vibrations are transformed into noise voltage and current due to the relative motion between the capacitive plates as well as the piezoelectric nature of many modern ceramic dielectrics. The piezoelectric effect is highly dependent on the dialectic material and on the capacitor construction. Capacitor manufacturer have the best information about the piezoelectric properties of their products and should be consulted for more specific information.

Inductors

The noise voltage from an inductor can be found from exploring the series combination of an inductor and a resistor.

$$|Z| = |R_s + j\omega L| = \sqrt{{R_s}^2 + \omega^2 L^2}\ .$$

$$e_{nL} = \sqrt{4KTB\ \boldsymbol{Re}(\boldsymbol{Z})} = \sqrt{4KTBR_s}$$

The power delivered by an inductor to a load is

$$P_{nL} = \frac{|e_{nLoad}|^2}{R_L} = \frac{{e_{nL}}^2}{R_L}\frac{{R_L}^2}{\omega^2 L^2 + (R_s + R_L)^2} = \frac{4KTBR_s}{R_L}\ \ \frac{{R_L}^2}{\omega^2 L^2 + (R_s + R_L)^2}$$

$$P_{nL} = 4KTBR_s\ \ \frac{R_L}{\omega^2 L^2 + (R_s + R_L)^2}$$

This is a low pass function.

If $\omega \ll \frac{R_L}{L}$ and $R_s \ll R_L$

$$P_{nL} = 4KTBR_s\ \ \frac{R_L}{(R_s + R_L)^2} = 4KTB\,\frac{R_s}{R_L}$$

The above equation indicates the noise from inductors is always much less than KTB.

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7 (Nelson & Davidson, 2002)

Appendix 2: Shot Noise

While it is often appropriate to consider electrical current as a continuous quantity, it must be kept in mind that it is composed of discrete electrons, each with a fixed charge. Current is quantized. Rather than measuring a continuous flow, the measurement of current is composed of sensing the effects of individual electrons arriving at a detector with a uniform distribution over time. The effect of this variation in time of arrival is called *shot noise* or *corpuscular noise*.

Computing Shot Noise

Derivation assuming a Poisson Distribution

A measurement of electric current can be considered to be the product of the number of electrons, $n_{e,}$ that arrive at a load per second of time, T, multiplied by the charge, q_e of each electron.

$$I = \frac{n_e\, q_e}{T},$$

Where q_e refers to the charge on an electron.

q_e =1.60217646 × 10^{-19} coulombs

The time of arrival of the electrons follows a Poisson distribution. The Poisson distributions describes the behavior of events that occur with a constant probity over time. It states that the probability that there are K occurrences of an event given the expected value of that event λ is given by

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}.$$

It can be shown that the Poisson distribution has a mean value of λ and a variance of λ.

Let I be the average current. The mean or average value of the number of electron arrivals per second is $\lambda = \frac{I}{q_e}$.

The variance in the number of arrivals during an interval of time T is λ. The standard deviation of the number of arrivals per second is $\sqrt{\frac{\lambda}{T}}$.

The standard deviation of current is then

$$\sigma_i = q_e\sqrt{\frac{\lambda}{T}}.$$

The equivalent low-pass filter for an observation time of T seconds is (see appendix 3).

$$B = \frac{1}{2T}$$

$T = \frac{1}{2B}$, where B determines the low-pass frequency of an equivalent filter.

$$\sigma_i = q_e\sqrt{2f_1\lambda} = q_e\sqrt{2f_1\frac{I}{q_e}} = \sqrt{2BIq_e}$$

The equation above states that the variance or power of the current fluctuations are proportional to bandwidth and that shot noise has a flat power spectral density.

Appendix 3: Bandwidth Equivalent of Averaging

Consider a zero-mean white Gaussian random process$x(t)$ that is band limited to ω_o and has a variance ${\sigma_0}^2$. $x(t)$ has a Fourier transform $X(\omega)$.

$$\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|x(t)|^2\,dt = {\sigma_0}^2 = \int_{-\omega_o}^{\omega_o}|X(\omega)|^2 d\omega$$

The random process is white between the integration limits.

$$\int_{-\omega_o}^{\omega_o}|X(\omega)|^2\ d\omega = 2\omega_o|X(\omega)|^2 = {\sigma_0}^2$$

The flat power spectral density between the integration limits can be computed.

$$|X(\omega)|^2 = \frac{{\sigma_0}^2}{2\omega_o}$$

We now need to extend the above equations for the case where the time integration limits are finite. The finite integration limits are equivalent to multiplying $x(t)$ by a rectangular function with unity value in the interval $-\frac{T}{2} < t < \frac{T}{2}$. This is the equivalent of convolving the frequency domain function with a sinc function.

$$\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|x(t)|^2\,dt = \int_{-\infty}^{\infty}\left|X(\omega) * T\ sinc\ \frac{\omega T}{2\pi}\right|^2 d\omega$$

$$\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|x(t)|^2\,dt = T^2\ |X(\omega)|^2\int_{-\infty}^{\infty} sinc^2\ \frac{\omega T}{2\pi}d\omega$$

$$\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|x(t)|^2\,dt = T^2\ |X(\omega)|^2\frac{2\pi}{T} = \pi\ \frac{{\sigma_0}^2}{\omega_o}$$

We can now compute the equivalent low-pass filter to the averaging process

$$\pi\ \frac{{\sigma_0}^2}{\omega_o} = \int_{-\omega_1}^{\omega_1}\frac{{\sigma_0}^2}{2\omega_o}d\omega = {\sigma_0}^2\frac{T\ \omega_1}{\omega_o}$$

$$\omega_1 = \frac{\pi}{T}$$

$$f_1 = \frac{1}{2T}$$

Appendix 4: Avalanche Diode Noise Sources

Avalanche diodes are often used as the source of excess noise. Diodes when biased into the avalanche breakdown region produce significant amounts of excess noise with good stability, high bandwidth, and good statistical properties. These noise sources are often used as the stimulus for noise figure measurements. They are also used as convenient sources of wideband random signals.

Avalanche Breakdown

Avalanche breakdown occurs due to the relatively high potentials that can exist across a revers biased junction. An electron can accelerate to very high speeds across this potential. A fast electron, when it strikes the semiconductor crystal lattice, can dislodge an electron from atoms in the crystal, creating a hole-electron pair. The free electron can in turn be accelerated, generating another hole-electron pair when it strikes the crystal lattice. This process can go on, effectively multiplying the current that flows.

It is useful to think of each electron that flows as having a multiplication factor, M.

$$M = \frac{1}{1-\left|\frac{V_D}{V_B}\right|^{\alpha}} \; .$$

α in the above equation is an exponent that varies between 3 and 6 depending on the semiconductor characteristics, V_B is the diode breakdown voltage and V_D is the actual voltage across the diode.

Consider a diode whose I-V characteristics are given by the familiar diode equation

$$I = I_0\left(e^{\frac{qV}{KT}} - 1\right)$$

When the diode is reverse biased with the application of a large negative voltage, the current is given by

$$I_R = I_0\left(1 - e^{-\frac{qV_R}{KT}}\right) \approx I_0$$

Reverse breakdown happens when the reverse bias current is multiplied

$$I_R = I_0 M = \frac{I_0}{1-\left|\frac{V_D}{V_B}\right|^{\alpha}}$$

The net effect is that a large amount of current can flow, creating the familiar diode breakdown curve shown in Figure 4.1

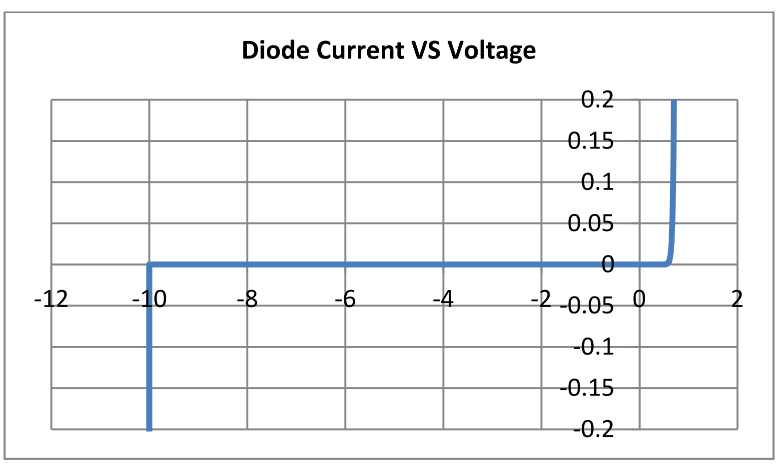

Figure 4.1. Diode with a 10 V breakdown.

Noise Mechanism

Like shot noise, current comes in discrete quantities. Unlike shot noise, the size of the discrete quantity is much larger than one electron, since each electron that crosses the potential barrier has liberated several others to flow. Each electron that flows is very quickly followed by a large number others, creating a larger granularity to the current flow.

A useful insight is obtained by thinking of the current flow as composed of discrete quantities that are much larger than one electron. We can then modify the equation for shot noise and apply a multiplier, β, to the magnitude of each discrete charge. This multiplier accounts for the fact that each time an electron travels, it brings many others with it.

$$\sigma_i = \sqrt{2f_1 I \, \beta q_e}$$

A Practical Noise Source

One of the useful properties of noise diodes is that their noise power is relatively independent of temperature, depending only on the level of current, which is easily controlled.

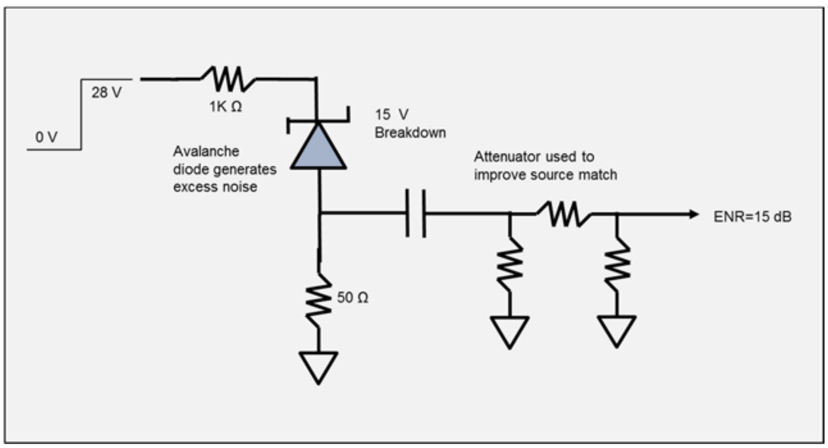

Figure 4.2. Noise source that can be switched from a cold to a hot state.

Figure 4.1 shows an example of a noise source that can be used for Y-factor noise measurements. A 28V control signal is used to switch the diode from cold to hot state. The Diode produces no excess noise when the control signal is at 0 V. Switching the control signal to 28V produces excess noise. The resistive attenuator is used to set the correct Excess Noise Ratio (ENR) as well as to improve the source match presented to the device under test.

The presence of an attenuator is the reason that sources with high ENR typically have worse reflection coefficients. Lower values of ENR with very good match can be achieved by combining a noise source with a high value of ENR and resistive attenuator.

Appendix 5: Error Analysis of the Y-factor Method

Y-factor Measurements

The Y-factor is given by making two power measurements. One measurement is made with an input excess noise source turned on, the other with the noise source turned off.

$\boldsymbol{Y} = \frac{\boldsymbol{P_{on}}}{\boldsymbol{P_{off}}}$, where P_{on} and P_{off} are power measurements made with the noise source on and off respectively.

The noise factor of a device can be computed from

$$F = \frac{\boldsymbol{ENR}}{\boldsymbol{Y-1}} = \frac{\boldsymbol{ENR}}{\frac{\boldsymbol{P_{on}}}{\boldsymbol{P_{off}}}-\boldsymbol{1}} = \boldsymbol{ENR}\frac{\boldsymbol{P_{off}}}{\boldsymbol{P_{on}-P_{off}}}$$

Noise Factor of a Measurement Receiver

The noise factor of the test receiver is made by making two power measurements with the noise source connected directly to the receiver.

$\boldsymbol{F_R} = \frac{\boldsymbol{ENR-1}}{\boldsymbol{Y-1}} = (\boldsymbol{ENR}-\boldsymbol{1})\frac{\boldsymbol{P_{off}}}{\boldsymbol{P_{on}-P_{off}}} = (\boldsymbol{ENR}-\boldsymbol{1})\frac{\boldsymbol{P_1}}{\boldsymbol{P_2-P_1}}$, where P_1 is a power measurements made with the noise source in the off or cold state directly connected to the measuring receiver, and P_2 is the same power measurement made with the noise source turned on.

Noise Factor of the DUT Cascaded with the Measurement Receiver

The noise factor of the DUT can only be measured as the cascade of the DUT and the measuring device.

$\boldsymbol{F_C} = (\boldsymbol{ENR}-\boldsymbol{1})\frac{\boldsymbol{P_3}}{\boldsymbol{P_4-P_3}}$, where P_3 is a power measurements made at the DUT output with the excess noise source in the off or cold state connected to the DUT input, and P_4 is the same power measurement made with the noise source turned on .

The cascaded noise factor can be used to compute the DUT noise factor using the Friis[8] formula.

$$\boldsymbol{F_C} = \boldsymbol{F_{DUT}} + \frac{\boldsymbol{F_R}-\boldsymbol{1}}{\boldsymbol{G_{DUT}}}$$

$$\boldsymbol{F_{DUT}} = \boldsymbol{F_c} - \frac{\boldsymbol{F_R}-\boldsymbol{1}}{\boldsymbol{G_{DUT}}}$$

The power gain of the device under test can be computed from the power measurements using

$$\boldsymbol{G_{DUT}} = \frac{\boldsymbol{P_4}-\boldsymbol{P_3}}{\boldsymbol{P_2}-\boldsymbol{P_1}}$$

The DUT noise factor can be computed using the power measurements made above

$$\boldsymbol{F_{DUT}} = \frac{(\boldsymbol{ENR_{spec}}-\boldsymbol{1})(\boldsymbol{P_3}-\boldsymbol{P_1})+\boldsymbol{P_2}-\boldsymbol{P_1}}{\boldsymbol{P_4}-\boldsymbol{P_3}}$$

The subscript in $\mathrm{ENR_{spec}}$ refers to the value of ENR that is supplied with the ENR source. It is used to differentiate from the actual value of ENR that is a contributor to the measurements of $\mathrm{P_2}$ and $\mathrm{P_4}$.

[8] (Friis, 1944)

Uncertainty in Noise Factor Measurements

The uncertainty in measuring the Noise factor of the DUT can be computed from the uncertainties in making the four power measurements.

Sources of Errors in Power Measurements

The noise factor of a device under test is computed from four power measurements as well as prior knowledge of the Excess Noise Ratio Errors of a noise source. Errors can come from all 5 terms in the equation.

Errors in measuringP_1, P_2, $P_3$and P_4 can be thought of as errors in measuring an incident power.

$$\boldsymbol{P_1 = kTB\left(1 \pm \Delta_{Gain} \pm \Delta_{lin} \pm \Delta_{\rho\ N-R}\right) + \ N_{AR}}$$

$$\boldsymbol{P_2 = \ ENR\ kTB\left(1 \pm \Delta_{Gain} \pm \Delta_{lin} \pm \Delta_{\rho\ N-R}\right) + \ N_{AR}}$$

Errors in measuring $\boldsymbol{P_3}$and $\boldsymbol{P_4}$

$$\boldsymbol{P_3 = (G_{DUT}\ KTB + N_{ADUT})\left(1 \pm \Delta_{Gain} \pm \Delta_{lin} \pm \Delta_{\rho\ N-DUT} \pm \Delta_{\rho\ DUT-R}\right) + \ N_{AR}}$$

$$\boldsymbol{P_4 = (G_{DUT}ENR\ \ KTB + N_{ADUT})\left(1 \pm \Delta_{Gain} \pm \Delta_{lin} \pm \Delta_{\rho\ N-DUT} \pm \Delta_{\rho\ DUT-R}\right) + \ N_{AR}}$$

Where:

Δ_{Gain} is the measuring receiver gain error. This absolute error includes calibration error, frequency response, temperature dependence, etc. This error is common to all power measurements used in measuring noise figure and gain, and experiences cancellation in ratio measurements such as the ones used in determining noise figure and gain. The receiver gain error is common to all measurement and is a function of frequency.

There are two cases to consider:

1. Cases where all power measurements are done at the same frequency have a common gain error.
2. Frequency response can cause the gain error to be different when measurements are made at different frequencies. This may be the case with frequency converters.

Δ_{lin} is the linearity of the receiver over its dynamic range. This includes the linearity of any amplifiers in the signal path as well as quantization errors of internal ADCs.

$\boldsymbol{N_{AR}}$ refers to the amount of noise power added by the receiver. It will also cause variations in the power measurement result caused by receiver noise. The variance in the power measurement caused by noise can be reduced to an arbitrarily small value by averaging and can be assumed to be zero for large numbers of averages.

$\boldsymbol{N_{ADUT}}$ refers to the noise added by the device under test. The mean value of this additive error is accounted for in the power measurements outlined in this pepper. Averaging can make the variations arbitrarily small.

$\Delta_{\rho\, N-R}$ is the error caused by the mismatch in the impedances of the noise source and the receiver. The mismatch error between the noise source and the receiver is given by

$$\Delta_{\rho\, N-R} = \mathbf{1} - \frac{\mathbf{1}}{|\mathbf{1} \pm \boldsymbol{\rho_N}\, \boldsymbol{\rho_R}|^2} \approx \pm\mathbf{2}\, \boldsymbol{\rho_N}\, \boldsymbol{\rho_R}$$

$\Delta_{\rho\, N-DUT}$ is the error caused by the mismatch in the impedances of the noise source and device under test.

$$\Delta_{\rho\, N-DUT} = \mathbf{1} - \frac{\mathbf{1}}{|\mathbf{1} \pm \boldsymbol{\rho_N}\, \boldsymbol{\rho_{DUTin}}|^2} \approx \pm\mathbf{2}\, \boldsymbol{\rho_N}\, \boldsymbol{\rho_{DUTin}}$$

$\Delta_{\rho\, DUT-R}$ is the error caused by the mismatch in the impedances of the DUT and the receiver. The mismatch error between the DUT and the receiver is given by

$$\Delta_{\rho\, DUT-R} = \mathbf{1} - \frac{\mathbf{1}}{|\mathbf{1} \pm \boldsymbol{\rho_{DUTout}}\, \boldsymbol{\rho_R}|^2} \approx \pm\mathbf{2}\, \boldsymbol{\rho_{DUTout}}\, \boldsymbol{\rho_R}$$

There are three cases to consider:

1. **The noise source reflection coefficient is the same in both the hot and cold state. This is often the case with noise diodes with a low value of ENR.** The error is correlated between measurements of P1 and P2 when this is the case. The mismatch error is also correlated between measurements of P3 and P4 when the noise source reflection coefficient does not change between the cold and hot state.
2. **The noise source reflection coefficient changes between the cold and hot state and the DUT has enough isolation so that its output reflection coefficient is independent of the source connected to its input.** The error is uncorrelated between P1 and P2 but correlated between P3 and p4.
3. **The noise source reflection coefficient changes between the cold and hot state and the DUT has low isolation so that its output reflection coefficient changes with changes in the source connected to its input**. All mismatch errors are not correlated and should be treated as independent.

Computing the Sensitivities for $\boldsymbol{F_{DUT}}$

$$\boldsymbol{F_{DUT}} = \frac{(ENR_{spec} - 1)(\boldsymbol{P_3} - \boldsymbol{P_1}) + \boldsymbol{P_2} - \boldsymbol{P_1}}{\boldsymbol{P_4} - \boldsymbol{P_3}}$$

The sensitivities of the noise factor measurement to errors in each of the power measurements can be computed by taking partial derivatives of the noise factor with respect to each of the four power measurements.

Noise Figure Sensitivity to Errors in Measuring $\boldsymbol{P_1}$

$$\frac{\partial F_{DUT}}{\partial P_1} = -\frac{ENR}{P_4 - P_3}$$

$$P_4 - P_3 = G_{DUT} ENR\ kTB + N_{ADUT} + N_{AR} - (G_{DUT}\ kTB + N_{ADUT} + N_{AR}) = (ENR - 1)G_{DUT}\ kTB$$

$$\frac{\delta F_{DUT}}{\delta P_1} = -\frac{ENR}{(ENR - 1)G_{DUT} kTB}$$

Noise Figure Sensitivity to Errors in Measuring $\boldsymbol{P_2}$

$$\frac{\delta F_{DUT}}{\delta P_1} = -\frac{ENR}{(ENR - 1)G_{DUT} kTB}$$

$$\frac{\delta F_{DUT}}{\delta P_2} = \frac{1}{(ENR - 1)G_{DUT} kTB}$$

Noise Figure Sensitivity to Errors in Measuring P_3

$$\frac{\partial F_{DUT}}{\partial P_3} = \frac{(ENR-1)P_4 - ENRP_1 + P_2}{(P_4 - P_3)^2}$$

$$\frac{\partial F_{DUT}}{\partial P_3} = \frac{(ENR-1)(G_{DUT}ENR\ kTB + N_{ADUT} + N_{AR}) - ENR(\ kTB + N_{AR}) + ENR\ kTB + N_{AR}}{(G_{DUT}ENR\ kTB + N_{ADUT} + N_{AR} - G_{DUT}\ kTB + N_{ADUT} - N_{AR})^2}$$

$$\frac{\partial F_{DUT}}{\partial P_3} = \frac{ENR + F_{DUT} - 1}{(ENR-1)G_{DUT}\ kTB}$$

$$\boldsymbol{\frac{\partial F_{DUT}}{\partial P_3} = \frac{1}{G_{DUT}\ kTB}\left(1 + \frac{F_{DUT}}{(ENR-1)}\right)}$$

Noise Figure Sensitivity to Errors in Measuring P_4

$$\frac{\partial F_{DUT}}{\partial P_4} = -\frac{(ENR-1)\ P_3 - ENRP_1 + P_2}{(P_4 - P_3)^2}$$

$$\frac{\partial F_{DUT}}{\partial P_4} = -\frac{(ENR-1)(\mathrm{G}_{DUT}kTB + N_{ADUT} + N_{AR}) - ENR(kTB + N_{AR}) + ENR\ kTB + N_{AR}}{(ENR-1)^2(\ G_{DUT}\ kTB)^2}$$

$$\frac{\partial F_{DUT}}{\partial P_4} = -\frac{G_{DUT}\ kTB + G_{DUT}\ kTB(F_{DUT} - 1)}{(ENR-1)(G_{DUT}\ kTB)^2}$$

$$\boldsymbol{\frac{\partial F_{DUT}}{\partial P_4} = -\frac{F_{DUT}}{(ENR-1)G_{DUT}\ kTB}}$$

Sensitivity Table

Table 5-1 lists the sensitivity of the noise factor measurement to errors in each of the four power measurements.

$$\boldsymbol{F_{DUT}} = \frac{(ENR_{spec} - 1)(\boldsymbol{P_3} - \boldsymbol{P_1}) + \boldsymbol{P_2} - \boldsymbol{P_1}}{\boldsymbol{P_4} - \boldsymbol{P_3}}$$

Sensitivity		Equation
$\frac{\partial \boldsymbol{F_{DUT}}}{\partial \boldsymbol{P_1}}$	$-\frac{ENR}{P_4 - P_3}$	$-\frac{ENR}{(\boldsymbol{ENR} - \boldsymbol{1})\boldsymbol{G_{DUT}}\, \boldsymbol{kTB}}$
$\frac{\partial \boldsymbol{F_{DUT}}}{\partial \boldsymbol{P_2}}$	$\frac{1}{P_4 - P_3}$	$\frac{1}{(\boldsymbol{ENR} - \boldsymbol{1})\boldsymbol{G_{DUT}}\, \boldsymbol{kTB}}$
$\frac{\partial \boldsymbol{F_{DUT}}}{\partial \boldsymbol{P_3}}$	$\frac{(ENR - 1)P_4 - ENRP_1 + P_2}{(P_4 - P_3)^2}$	$\frac{\boldsymbol{1}}{\boldsymbol{G_{DUT}}\, \boldsymbol{kTB}}\left(1 + \frac{\boldsymbol{F_{DUT}}}{(\boldsymbol{ENR} - \boldsymbol{1})}\right)$
$\frac{\partial \boldsymbol{F_{DUT}}}{\partial \boldsymbol{P_4}}$	$-\frac{(ENR - 1)\, P_3 - ENRP_1 + P_2}{(P_4 - P_3)^2}$	$-\frac{\boldsymbol{F_{DUT}}}{\boldsymbol{G_{DUT}}\, \boldsymbol{kTB}(\boldsymbol{ENR} - \boldsymbol{1})}$

Table 5-1. Sensitivities to power measurement errors.

We must now find the various sources of power measurement errors and compute how they propagate to the measurement of a device’s noise factor.

Sensitivities to Measurement Errors

We can now determine the sensitivity of the noise factor measurement to each of the sources of error.

Sensitivity to Measuring Instrument Noise Power Subtraction Uncertainty

The equation for noise factor is

$$\boldsymbol{F_{DUT}} = \frac{(ENR_{spec} - 1)(\boldsymbol{P_3} - \boldsymbol{P_1}) + \boldsymbol{P_2} - \boldsymbol{P_1}}{\boldsymbol{P_4} - \boldsymbol{P_3}}$$

The measurements of noise factor and gain depend on the ability to accurately subtract power levels made at relatively low levels. The quantity $\boldsymbol{P_1}$ is effectively a measure of the measuring instrument's noise floor. $\boldsymbol{P_2}$ is a measure of the power from the noise source, typically 5 to 15 dB higher than the kTB. $\boldsymbol{P_3}$ abd $\boldsymbol{P_4}$ are power measurements made with the DUT in place and the noise source turned off and on respectively.

Most measuring instruments have limitations on how accurately two power measurements can be subtracted from each other. These include:

1. Linearity
2. ADC Quantization floor
3. Quantization errors
4. Spurious signals

The receiver has a minimum difference between two powers that it can accurately resolve then. Let this power subtraction uncertainty be equal to P_u .

$$\boldsymbol{F_{DUT}} = \frac{(ENR_{spec} - 1)(\boldsymbol{P_3} - \boldsymbol{P_1} + P_u) + \boldsymbol{P_2} - \boldsymbol{P_1} + P_u}{\boldsymbol{P_4} - \boldsymbol{P_3} + P_m}$$

$$\frac{(ENR_{spec} - 1)(\boldsymbol{P_3} - \boldsymbol{P_1} +) + \boldsymbol{P_2} - \boldsymbol{P_1} + ENR_{spec}P_u}{\boldsymbol{P_4} - \boldsymbol{P_3} + P_u}$$

The sensitivity of the noise figure measurement to the minimum power is then

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial P_u} = \frac{ENR_{spec}(\boldsymbol{P_4} - \boldsymbol{P_3} + P_u) - (ENR_{spec} - 1)(\boldsymbol{P_3} - \boldsymbol{P_1} +) + \boldsymbol{P_2} - \boldsymbol{P_1} + ENR_{spec} P_u}{(\boldsymbol{P_4} - \boldsymbol{P_3} + P_u)^2}$$

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial P_m} = \frac{ENR_{spec}}{\boldsymbol{P_4} - \boldsymbol{P_3} + P_u} - \frac{(ENR_{spec} - 1)(\boldsymbol{P_3} - \boldsymbol{P_1} +) + \boldsymbol{P_2} - \boldsymbol{P_1}}{\boldsymbol{P_4} - \boldsymbol{P_3}} \frac{\boldsymbol{P_4} - \boldsymbol{P_3}}{(\boldsymbol{P_4} - \boldsymbol{P_3} + P_u)^2} - \frac{ENR_{spec} P_u}{(\boldsymbol{P_4} - \boldsymbol{P_3} + P_u)^2}$$

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial P_m} = \frac{ENR_{spec}}{\boldsymbol{P_4} - \boldsymbol{P_3} + P_m} - \boldsymbol{F_{DUT}} \frac{\boldsymbol{P_4} - \boldsymbol{P_3}}{(\boldsymbol{P_4} - \boldsymbol{P_3} + P_m)^2} - \frac{ENR_{spec} P_u}{(\boldsymbol{P_4} - \boldsymbol{P_3} + P_m)^2}$$

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial P_m} = \frac{ENR_{spec}}{(ENR - 1)G_{DUT}\, kTB + P_u} - \boldsymbol{F_{DUT}} \frac{(ENR - 1)G_{DUT}\, kTB + ENR_{spec} P_u}{\left((ENR - 1)G_{DUT}\, kTB + P_u\right)^2}$$

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial P_m} = \frac{ENR_{spec}}{(ENR - 1)G_{DUT}\, kTB + P_u} - \boldsymbol{F_{DUT}} \frac{(ENR - 1)G_{DUT}\, kTB + P_u + (ENR_{spec} - 1)P_u}{\left((ENR - 1)G_{DUT}\, kTB + P_u\right)^2}$$

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial P_m} = \frac{ENR_{spec} - \boldsymbol{F_{DUT}}}{(ENR - 1)G_{DUT}\, kTB + P_u} - \boldsymbol{F_{DUT}} \frac{(ENR_{spec} - 1)u}{\left((ENR - 1)G_{DUT}\, kTB + P_u\right)^2}$$

Let M be the power subtraction floor expressed as a ratio to KTB. We can also assume that the ENR specification error is small.

$$P_u = M\, kTB, and\ \ ENR_{spec} = ENR$$

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial M} = \frac{ENR - \boldsymbol{F_{DUT}}}{(ENR - 1)G_{DUT}\ + M} - \boldsymbol{F_{DUT}} \frac{(ENR - 1)M}{\left[(ENR - 1)G_{DUT}\ + M\ \right]^2}$$

Sensitivity to ENR Accuracy Errors

ENR accuracy is defined as the error between the specified value of ENR and the actual value.

$$ENR_{Spec} = ENR(1 \pm \Delta_{ENR})$$

The equation for noise factor then becomes

$$\boldsymbol{F_{DUT}} = \frac{[ENR(1 \pm \Delta_{ENR}) - 1](\boldsymbol{P_3} - \boldsymbol{P_1}) + \boldsymbol{P_2} - \boldsymbol{P_1}}{\boldsymbol{P_4} - \boldsymbol{P_3}} \ .$$

The sensitivity to ENR errors is compute from the partial derivatives,

$$\frac{\partial F_{DUT}}{\partial \Delta_{ENR}} = ENR \frac{\boldsymbol{P_3} - \boldsymbol{P_1}}{\boldsymbol{P_4} - \boldsymbol{P_3}} + \frac{\partial F_{DUT}}{\partial \boldsymbol{P_1}} \frac{\partial \boldsymbol{P_1}}{\partial \Delta_{ENR}} + \frac{\partial F_{DUT}}{\partial \boldsymbol{P_2}} \frac{\partial \boldsymbol{P_2}}{\partial \Delta_{ENR}} + \frac{\partial F_{DUT}}{\partial \boldsymbol{P_3}} \frac{\partial \boldsymbol{P_3}}{\partial \Delta_{ENR}} + \frac{\partial F_{DUT}}{\partial \boldsymbol{P_4}} \frac{\partial \boldsymbol{P_4}}{\partial \Delta_{ENR}} \ .$$

We can see from the formulas for the four power measurements that they have a dependence on the actual value of ENR, but not on the error in specifying it.

$$\frac{\partial P_1}{\partial \Delta_{ENR}} = \frac{\partial P_2}{\partial \Delta_{ENR}} = \frac{\partial P_3}{\partial \Delta_{ENR}} = \frac{\partial P_4}{\partial \Delta_{ENR}} = 0$$

$$\frac{\partial F_{DUT}}{\partial \Delta_{ENR}} = ENR \frac{P_3 - P_1}{P_4 - P_3}$$

We can now expand the components of the four power measurements.

$$\frac{\partial F_{DUT}}{\partial \Delta_{ENR}} = ENR \frac{\mathrm{kTB}G_{DUT} + N_{ADUT} + N_{AR} - kTB - N_{AR}}{\mathrm{ENR}\ \mathrm{kTB}G_{DUT} + N_{ADUT} + N_{AR} - \mathrm{kTB}G_{DUT} - N_{ADUT} - N_{AR}}$$

$$\frac{\partial F_{DUT}}{\partial \Delta_{ENR}} = ENR \frac{\mathrm{kTB}G_{DUT} + (F_{DUT} - 1)kTB\ G_{DUT} - kTB}{(ENR - 1)G_{DUT}\ kTB}$$

$$\boldsymbol{\frac{\partial F_{DUT}}{\partial \Delta_{ENR}} = \frac{ENR}{(ENR - 1)} \frac{F_{DUT}\ G_{DUT} - 1}{G_{DUT}}}$$

Sensitivity to Receiver Gain Errors

For the dependence on the receiver gain we must recognize that the gain is common to all four power measurements. There is cancellation as the gain used in measuring P_2 subtracts from the gain used in measuringP_1, and so forth. This means that any receiver gain (measurement accuracy) that is common to all power measurements does not affect the measurement of noise figure.

We must recognize, however, that the four power measurements are not made simultaneously and there may be seconds or even minutes between the power measurements. The measurements may not be at the same frequency in the case of frequency converters. Minute changes in gain as the four power measurements are made can affect noise figure measurements. These changes can be:

- Gain drift over time
- Gain drift as the environmental temperature changes
- Frequency response if P_1 and P_2 are made at different frequencies than P_3 and P_4.

Let Δ_{Gain} be any change in gain that can happen between during the four measurements.

$$\left(\frac{\partial F_{DUT}}{\partial \Delta_{Gain}}\right)^2 = \left(\frac{\partial F_{DUT}}{\partial P_1}\,\frac{\partial P_1}{\partial \Delta_{Gain}}\right)^2 + \left(\frac{\partial F_{DUT}}{\partial P_2}\,\frac{\partial P_2}{\partial \Delta_{Gain}}\right)^2 + \left(\frac{\partial F_{DUT}}{\partial P_{13}}\,\frac{\partial P_3}{\partial \Delta_{Gain}}\right)^2 + \left(\frac{\partial F_{DUT}}{\partial P_4}\,\frac{\partial P_4}{\partial \Delta_{Gain}}\right)^2$$

$$\begin{aligned}\left(\frac{\partial F_{DUT}}{\partial \Delta_{Gain}}\right)^2 = & \left(-\frac{ENR}{(ENR-1)G_{DUT}\,kTB}\,kTB\right)^2 + \left(\frac{1}{(ENR-1)G_{DUT}\,kTB}\,ENR\,KTB\right)^2 \\ & + \left[\frac{1}{G_{DUT}\,kTB}\left(1+\frac{F_{DUT}}{(ENR-1)}\right)(G_{DUT}\,kTB + N_{ADUT})\right]^2 \\ & + \left[-\frac{F_{DUT}}{(ENR-1)G_{DUT}\,kTB}\,(ENR\,G_{DUT}\,kTB + N_{ADUT})\right]^2\end{aligned}$$

The noise added by the device under test can be expressed as $N_{ADUT} = (F_{DUT}-1)G_{DUT}\,kTB$. Substitution yields:

$$\begin{aligned}\left(\frac{\partial F_{DUT}}{\partial \Delta_{Gain}}\right)^2 = & \left(-\frac{ENR}{(ENR-1)G_{DUT}\,kTB}\,kTB\right)^2 + \left(\frac{1}{(ENR-1)G_{DUT}\,kTB}\,ENR\,KTB\right)^2 \\ & + \left[\frac{1}{G_{DUT}\,kTB}\left(1+\frac{F_{DUT}}{(ENR-1)}\right)(G_{DUT}\,kTB + (F_{DUT}-1)G_{DUT}\,kTB)\right]^2 \\ & + \left[-\frac{F_{DUT}}{(ENR-1)G_{DUT}\,kTB}\,(ENR\,G_{DUT}\,kTB + (F_{DUT}-1)G_{DUT}\,kTB)\right]^2\end{aligned}$$

$$\begin{aligned}\left(\frac{\partial F_{DUT}}{\partial \Delta_{Gain}}\right)^2 = & \left(-\frac{ENR}{(ENR-1)G_{DUT}}\right)^2 + \left(\frac{ENR}{(ENR-1)G_{DUT}}\right)^2 + \left[\left(1+\frac{F_{DUT}}{(ENR-1)}\right)F_{DUT}\right]^2 \\ & + \left[-\frac{F_{DUT}}{(ENR-1)}\,(ENR\ + (F_{DUT}-1)\,)\right]^2\end{aligned}$$

$$\left(\frac{\partial F_{DUT}}{\partial \Delta_{Gain}}\right)^2 = 2\left(\frac{ENR}{(ENR-1)G_{DUT}}\right)^2 + 2\,F_{DUT}{}^2\left(\frac{ENR-1+F_{DUT}}{(ENR-1)}\right)^2$$

$$\frac{\Delta F_{DUT}}{F_{DUT}} = \frac{\sqrt{2}\,\Delta_{Gain}}{(ENR-1)}\sqrt{\left(\frac{ENR}{G_{DUT}\,F_{DUT}}\right)^2 + (ENR-1+F_{DUT})^2}$$

Sensitivity to Linearity Errors

Linearity error is common to all four power measurements, but is not correlated. Its contributions need to be added in an RSS fashion.

$$\left(\frac{\partial F_{DUT}}{\partial \Delta_{Lin}}\right)^2 = \left(\frac{\partial F_{DUT}}{\partial P_1}\frac{\partial P_1}{\partial \Delta_{Lin}}\right)^2 + \left(\frac{\partial F_{DUT}}{\partial P_2}\frac{\partial P_2}{\partial \Delta_{Lin}}\right)^2 + \left(\frac{\partial F_{DUT}}{\partial P_{13}}\frac{\partial P_3}{\partial \Delta_{Lin}}\right)^2 + \left(\frac{\partial F_{DUT}}{\partial P_4}\frac{\partial P_4}{\partial \Delta_{Lin}}\right)^2$$

$$\begin{aligned}\left(\frac{\partial F_{DUT}}{\partial \Delta_{Lin}}\right)^2 = {} & \left(-\frac{ENR\ \mathrm{kTB}}{(ENR-1)\mathrm{kTB\ G}_{DUT}}\right)^2 + \left(\frac{ENR\ \mathrm{kTB}}{(ENR-1)\mathrm{kTB\ G}_{DUT}}\right)^2 \\ & + \left[\frac{1}{G_{DUT}\ kTB}\left(1+\frac{F_{DUT}}{(ENR-1)}\right)(G_{DUT}\ kTB + N_{ADUT})\right]^2 \\ & + \left[-\frac{F_{DUT}}{G_{DUT}\ kTB(ENR-1)}(ENR\ G_{DUT}\ kTB + N_{ADUT})\right]^2\end{aligned}$$

A derivation similar to the one above for gain variations yields the following

$$\frac{\Delta F_{DUT}}{F_{DUT}} = \frac{\sqrt{2}\,\Delta_{Lin}}{(ENR-1)}\sqrt{\left(\frac{ENR}{G_{DUT}\,F_{DUT}}\right)^2 + (ENR-1+F_{DUT})^2}$$

Sensitivity to Mismatch Errors

The sensitivity of the four power measurements to mismatches that exist during their measurement can be computed from the formulas for each of the four power quantities, reproduced here for convenience.

$$\boldsymbol{P_1 = kTB(1 \pm \Delta_{Gain} \pm \Delta_{lin} \pm \Delta_{\rho\, N-R}) + N_{AR}}$$

$$\boldsymbol{P_2 = ENR\ kTB(1 \pm \Delta_{Gain} \pm \Delta_{lin} \pm \Delta_{\rho\, N-R}) + N_{AR}}$$

$$\boldsymbol{P_3 = (1 \pm \Delta_{Gain} \pm \Delta_{lin} \pm \Delta_{\rho\, N-DUT} \pm \Delta_{\rho\, DUT-R}) + N_{AR}}$$

$$\boldsymbol{P_4 = (G_{DUT}ENR\ \ KTB + N_{ADUT})(1 \pm \Delta_{Gain} \pm \Delta_{lin} \pm \Delta_{\rho\, N-DUT} \pm \Delta_{\rho\, DUT-R}) + N_{AR}}$$

The sensitivities of the power measurements to each of the mismatch errors are:

$$\boldsymbol{\frac{\partial P_1}{\partial \Delta_{\rho 1}} = kTB}$$

$$\boldsymbol{\frac{\partial P_2}{\partial \Delta_{\rho 2}} = ENR\ kTB}$$

$$\boldsymbol{\frac{\partial P_3}{\partial \Delta_{\rho 3}} = G_{DUT}\ kTB + N_{ADUT}}$$

$$\boldsymbol{\frac{\partial P_4}{\partial \Delta_{\rho 4}} = G_{DUT}\ ENR\ kTB + N_{ADUT}}$$

The sensitivity of the noise factor computation to mismatch errors can be computed from:

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial \Delta_{\rho 1}} = \frac{\partial \boldsymbol{F_{DUT}}}{\partial \boldsymbol{P_1}} \frac{\partial \boldsymbol{P_1}}{\partial \Delta_{\rho 1}} = -\frac{\boldsymbol{ENR}}{(\boldsymbol{ENR}-\mathbf{1})\boldsymbol{G_{DUT}}}$$

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial \Delta_{\rho 2}} = \frac{\partial \boldsymbol{F_{DUT}}}{\partial \boldsymbol{P_2}} \frac{\partial \boldsymbol{P_2}}{\partial \Delta_{\rho 2}} = \frac{\boldsymbol{ENR}}{(\boldsymbol{ENR}-\mathbf{1})\boldsymbol{G_{DUT}}}$$

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial \Delta_{\rho 3}} = \frac{\partial \boldsymbol{F_{DUT}}}{\partial \boldsymbol{P_3}} \frac{\partial \boldsymbol{P_3}}{\partial \Delta_{\rho 1}} = \boldsymbol{F_{DUT}}\left[\frac{\boldsymbol{ENR}-\mathbf{1}+\boldsymbol{F_{DUT}}}{(\boldsymbol{ENR}-\mathbf{1})}\right] = \boldsymbol{F_{DUT}}\left[1+\frac{\boldsymbol{F_{DUT}}}{(\boldsymbol{ENR}-\mathbf{1})}\right]$$

$$\frac{\partial \boldsymbol{F_{DUT}}}{\partial \Delta_{\boldsymbol{\rho 4}}} = \frac{\partial \boldsymbol{F_{DUT}}}{\partial \boldsymbol{P_4}} \frac{\partial \mathbf{4}}{\partial \Delta_{\rho 4}} = -\boldsymbol{F_{DUT}}\left[\frac{\boldsymbol{ENR}+\mathbf{1}-\boldsymbol{F_{DUT}}}{(\boldsymbol{ENR}-\mathbf{1})}\right]$$

The mismatch error involved in each power measurement is:

$$\Delta_{\rho 1} = 1 - \frac{1}{\left(1 \pm \rho_{Noff}\,\rho_R\right)^2} \approx \pm 2\rho_{Noff}\,\rho_R$$

$$\Delta_{\rho 2} = 1 - \frac{1}{(1 \pm \rho_{Non}\,\rho_R)^2} \approx \pm 2\rho_{Non}\,\rho_R$$

$$\Delta_{\rho 3} = 1 - \frac{1}{\left(1 \pm \rho_{Noff}\,\rho_{DUTin}\right)^2}\frac{1}{(1 \pm \rho_{DUTout}\,\rho_R)^2} \approx \pm 2\,\rho_{Noff}\,\rho_{DUTin} \pm 2\,\rho_{DUTout}\,\rho_R$$

$$\Delta_{\rho 4} = 1 - \frac{1}{1 \pm \rho_{Non}\,\rho_{DUTin}}\frac{1}{1 \pm \rho_{DUTout}\,\rho_R} \approx \pm 2\,\rho_{Non}\,\rho_{DUTin} \pm 2\,\rho_{DUTout}\,\rho_R$$

The contributions from each of the mismatches can now be computed using a derivation similar to the one done for gain and linearity.

$$\Delta F_{DUT\ P1} = \frac{\partial F_{DUT}}{\partial \Delta_{\rho 1}}\Delta_{\rho 1} = -\frac{ENR}{(ENR-1)}\frac{2\rho_{Noff}\,\rho_R}{G_{DUT}}$$

$$\Delta F_{DUT\ P2} = \frac{\partial F_{DUT}}{\partial \Delta_{\rho 2}}\Delta_{\rho 2} = \frac{ENR}{(ENR-1)}\frac{2\rho_{Non}\,\rho_R}{G_{DUT}}$$

$$\Delta F_{DUT\ P3\ in} = \frac{\partial F_{DUT}}{\partial \Delta_{\rho 3}}\Delta_{\rho 3\ in} = F_{DUT}\left(1+\frac{F_{DUT}}{(ENR-1)}\right) 2\,\rho_{Noff}\,\rho_{DUTin}$$

$$\Delta F_{DUT\ P3\ out} = \frac{\partial F_{DUT}}{\partial \Delta_{\rho 3}}\Delta_{\rho 3\ out} = F_{DUT}\left(1+\frac{F_{DUT}}{(ENR-1)}\right) 2\,\rho_{DUTout}\,\rho_R$$

$$\Delta F_{DUT\ P4\ in} = \frac{\partial F_{DUT}}{\partial \Delta_{\rho 4}}\Delta_{\rho 4\ in} = -F_{DUT}\left[\frac{ENR+1-F_{DUT}}{(ENR-1)}\right] 2\,\rho_{Non}\,\rho_{DUTin}$$

$$\Delta F_{DUT\ P4\ out} = \frac{\partial F_{DUT}}{\partial \Delta_{\rho 4}}\Delta_{\rho 4\ out} = -F_{DUT}\left[\frac{ENR+1-F_{DUT}}{(ENR-1)}\right] 2\,\rho_{DUTout}\,\rho_R$$

Case 1: The noise source reflection coefficient does not change between the hot and cold state

If the noise source does not change its reflection coefficient between its hot and cold state, then the errors in measuring p1 and P2 are correlated, the errors in measuring p3 and p4 are correlated but the remaining errors from the two pairs of measurements are not correlated to each other.

$$\left(\Delta \boldsymbol{F}_{\boldsymbol{DUT\rho}}\right)^2 = (\Delta \boldsymbol{F}_{\boldsymbol{DUT\ P1}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P2}})^2 + (\Delta \boldsymbol{F}_{\boldsymbol{DUT\ P3\ in}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P4\ in}})^2 + (\Delta \boldsymbol{F}_{\boldsymbol{DUT\ P3\ out}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P4\ out}})^2$$

There is complete cancellation of the first term and partial cancellation of the other two.

$$\left(\Delta \boldsymbol{F}_{\boldsymbol{DUT\rho}}\right)^2 = (\Delta \boldsymbol{F}_{\boldsymbol{DUT\ P3\ in}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P4\ in}})^2 + (\Delta \boldsymbol{F}_{\boldsymbol{DUT\ P3\ out}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P4\ out}})^2$$

Case 2: The noise source reflection coefficient changes between its cold and hot state and the DUT has enough isolation so that its output reflection coefficient is independent of the noise source state.

In this case, power measurement errors due to mismatches at the source are uncorrelated and the contributions of the mismatch errors at the output are correlated.

$$\left(\Delta \boldsymbol{F}_{\boldsymbol{DUT\rho}}\right)^2 = \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P1}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P2}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P3\ in}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P4\ in}}{}^{\boldsymbol{2}} + (\Delta \boldsymbol{F}_{\boldsymbol{DUT\ P3\ out}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P4\ out}})^2$$

There is no cancellation of the errors in measuring p1 and P2, but there is partial cancellation in measuring the errors in P3 and P4

Case 3: The noise source reflection coefficient changes between its cold and hot state and the DUT poor isolation so that its output reflection coefficient depends on the noise source state.

In this case, power measurement errors due to mismatches at the source are uncorrelated and the contributions of the mismatch errors at the output are also uncorrelated.

$$\left(\Delta \boldsymbol{F}_{\boldsymbol{DUT\rho}}\right)^2 = \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P1}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P2}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P3\ in}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P4\ in}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P3\ out}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{F}_{\boldsymbol{DUT\ P4\ out}}{}^{\boldsymbol{2}}$$

Computing the Sensitivities for Device Gain

Sensitivity of G_{DUT} to power measurement errors

$$\Delta G_{DUT} = \frac{\partial G_{DUT}}{\partial P_1}\Delta P_1 + \frac{\partial G_{DUT}}{\partial P_2}\Delta P_2 + \frac{\partial G_{DUT}}{\partial P_3}\Delta P_3 + \frac{\partial G_{DUT}}{\partial P_4}\Delta P_4$$

$$G_{DUT} = \frac{P_4 - P_3}{P_2 - P_1}$$

$$P_1 = kTB + N_{AR}$$

$$P_2 = ENR\ KTB + N_{AR}$$

$$P_3 = G_{DUT}\ KTB + N_{ADUT} + N_{AR}$$

$$P_4 = G_{DUT} ENR\ \ KTB{+}N_{ADUT} + N_{AR}$$

$$\boldsymbol{\frac{\partial G_{DUT}}{\partial P_1} = \frac{P_4 - P_3}{(P_2 - P_1)^2} = \frac{(ENR-1)\mathrm{kTB}G_{DUT}}{(ENR-1)^2\ \mathrm{kTB}^2} = \frac{G_{DUT}}{(ENR-1)kTB}}$$

$$\boldsymbol{\frac{\partial G_{DUT}}{\partial P_2} = -\frac{P_4 - P_3}{(P_2 - P_1)^2} = -\frac{G_{DUT}}{(ENR-1)kTB}}$$

$$\boldsymbol{\frac{\partial G_{DUT}}{\partial P_3} = -\frac{1}{P_2 - P_1} = -\frac{1}{(ENR-1)\mathrm{kTB}}}$$

$$\boldsymbol{\frac{\partial G_{DUT}}{\partial P_4} = \frac{1}{P_2 - P_1} = \frac{1}{(ENR-1)kTB}}$$

Gain Sensitivity Table

Table 5-2 lists the sensitivity of the gain computation to errors in each of the four power measurements.

$$\boldsymbol{G_{DUT} = \frac{P_4 - P_3}{P_2 - P_1}}$$

Sensitivity	In Terms of Power	In Terms of ENR and G_{DUT}
$\boldsymbol{\frac{\partial G_{DUT}}{\partial P_1}}$	$\boldsymbol{\frac{P_4 - P_3}{(P_2 - P_1)^2}}$	$\boldsymbol{\frac{G_{DUT}}{(ENR-1)kTB}}$
$\boldsymbol{\frac{\partial G_{DUT}}{\partial P_2}}$	$\boldsymbol{-\frac{P_4 - P_3}{(P_2 - P_1)^2}}$	$\boldsymbol{-\frac{G_{DUT}}{(ENR-1)kTB}}$
$\boldsymbol{\frac{\partial G_{DUT}}{\partial P_3}}$	$\boldsymbol{-\frac{1}{P_2 - P_1}}$	$\boldsymbol{-\frac{1}{(ENR-1)\mathrm{kTB}}}$
$\boldsymbol{\frac{\partial G_{DUT}}{\partial P_4}}$	$\boldsymbol{\frac{1}{P_2 - P_1}}$	$\boldsymbol{\frac{1}{(ENR-1)kTB}}$

Table 5-2. Sensitivities to power measurement errors.

We must now examine the various sources of power measurement errors and compute how they propagate to the measurement of a device gain.

Computing the Gain Sensitivities to the Power Measurement Errors

Gain Sensitivity to the Noise Power Subtraction Uncertainty

The measurements of noise factor and gain depend on the ability to accurately subtract power levels made at relatively low levels. The quantity $\boldsymbol{P_1}$ is effectively a measure of the measuring instrument's noise floor. $\boldsymbol{P_2}$ is a measure of the power from the noise source, typically 5 to 15 dB higher than kTB. $\boldsymbol{P_3}$ abd $\boldsymbol{P_4}$ are power measurements made with the DUT in place and the noise source turned off and on respectively.

Most measuring instruments have limitations on how accurately two power measurements can be subtracted from each other. These include:

- Linearity
- ADC Quantization floor
- Quantization errors
- Spurious signals

$$\boldsymbol{G_{DUT}} = \frac{\boldsymbol{P_4} - \boldsymbol{P_3} + P_u}{\boldsymbol{P_2} - \boldsymbol{P_1} + P_u}$$

$$\frac{\boldsymbol{\partial\, G_{DUT}}}{\boldsymbol{\partial} P_u} = \frac{\boldsymbol{P_2} - \boldsymbol{P_1} + P_u - \boldsymbol{P_4} + \boldsymbol{P_3} - P_u}{(\boldsymbol{P_2} - \boldsymbol{P_1} + P_u)^{\mathbf{2}}}$$

$$\frac{\boldsymbol{\partial\, G_{DUT}}}{\boldsymbol{\partial} P_u} = \frac{\boldsymbol{P_2} - \boldsymbol{P_1} - \boldsymbol{P_4} + \boldsymbol{P_3}}{(\boldsymbol{P_2} - \boldsymbol{P_1} + P_u)^{\mathbf{2}}}$$

$$\frac{\boldsymbol{\partial\, G_{DUT}}}{\boldsymbol{\partial} P_u} = \frac{(\mathbf{ENR} - \mathbf{1})kTB - (\mathbf{ENR} - \mathbf{1})G_{DUT}\, kTB}{\left((\mathbf{ENR} - \mathbf{1})kTB + P_u\right)^{\mathbf{2}}}$$

$$\frac{\boldsymbol{\partial\, G_{DUT}}}{\boldsymbol{\partial} P_u} = \frac{(\mathbf{1} - G_{DUT})\,(\mathbf{ENR} - \mathbf{1})kTB}{\left((\mathbf{ENR} - \mathbf{1})kTB + P_u\right)^{\mathbf{2}}}$$

Let $P_u = M\ kTB$

$$\frac{\boldsymbol{\partial\, G_{DUT}}}{\boldsymbol{\partial} M} = -\frac{(G_{DUT} - 1)\,(\mathbf{ENR} - \mathbf{1})}{[(\mathbf{ENR} - \mathbf{1}) + M\,]^{\mathbf{2}}}$$

Device gain sensitivity to ENR Error

$$\frac{\partial G_{DUT}}{\partial \Delta_{ENR}} = \frac{\partial G_{DUT}}{\partial P_1}\,\frac{\partial P_1}{\partial \Delta_{ENR}} + \frac{\partial G_{DUT}}{\partial P_2}\,\frac{\partial P_2}{\partial \Delta_{ENR}} + \frac{\partial G_{DUT}}{\partial P_{13}}\,\frac{\partial P_3}{\partial \Delta_{ENR}} + \frac{\partial G_{DUT}}{\partial P_4}\,\frac{\partial P_4}{\partial \Delta_{ENR}}$$

$$\frac{\partial P_1}{\partial \Delta_{ENR}} = \frac{\partial P_2}{\partial \Delta_{ENR}} = \frac{\partial P_3}{\partial \Delta_{ENR}} = \frac{\partial P_4}{\partial \Delta_{ENR}} 0$$

$$\frac{\boldsymbol{\partial G_{DUT}}}{\boldsymbol{\partial \Delta_{ENR}}} = \mathbf{0}$$

Device Gain Sensitivity to Receiver Gain Errors

Gain errors are common to all power measurements, errors add algebraically and cancel. Any variation in gain that happens during the time that the power measurements are being made will show up as an error.

Let Δ_{Gain} Denote any gain variations that happen during the time the four measurements are made.

$$\left(\frac{\partial G_{DUT}}{\partial \Delta_{Gain}}\right)^2 = \left(\frac{\partial G_{DUT}}{\partial P_1}\frac{\partial P_1}{\partial \Delta_{Gain}}\right)^2 + \left(\frac{\partial G_{DUT}}{\partial P_2}\frac{\partial P_2}{\partial \Delta_{Gain}}\right)^2 + \left(\frac{\partial G_{DUT}}{\partial P_{13}}\frac{\partial P_3}{\partial \Delta_{Gain}}\right)^2 + \left(\frac{\partial G_{DUT}}{\partial P_4}\frac{\partial P_4}{\partial \Delta_{Gain}}\right)^2$$

$$\left(\frac{\partial G_{DUT}}{\partial \Delta_{Gain}}\right)^2 = \left(\frac{G_{DUT}}{(ENR-1)kTB}\,kTB\right)^2 + \left(-\frac{G_{DUT}}{(ENR-1)kTB}ENR\,kTB\right)^2 + \left(-\frac{1}{(ENR-1)\mathrm{kTB}}(G_{DUT}kTB + \boldsymbol{N_{ADUT}})\right)^2 + \left(\frac{1}{(ENR-1)\mathrm{kTB}}\;(ENRG_{DUT}kTB + \boldsymbol{N_{ADUT}})\right)^2$$

The noise added by the device under test can be expressed as $\boldsymbol{N_{ADUT}} = (\boldsymbol{F_{DUT}} - \mathbf{1})\boldsymbol{G_{DUT}}\,\boldsymbol{kTB}$. Substitution yields:

$$\left(\frac{\partial G_{DUT}}{\partial \Delta_{Gain}}\right)^2 = \left(\frac{G_{DUT}}{(ENR-1)}\right)^2 + \left(\frac{G_{DUT}\;ENR}{(ENR-1)}\right)^2 + \left(\frac{(G_{DUT} + (\boldsymbol{F_{DUT}} - \mathbf{1})\boldsymbol{G_{DUT}})}{(ENR-1)}\right)^2 + \left(\frac{ENRG_{DUT} + (\boldsymbol{F_{DUT}} - \mathbf{1})\boldsymbol{G_{DUT}}}{(ENR-1)}\right)^2$$

$$\left(\frac{\partial G_{DUT}}{\partial \Delta_{Gain}}\right)^2 = 2\left(\frac{G_{DUT}}{(ENR-1)}\right)^2\left[ENR^2 + \boldsymbol{F_{DUT}}^{\mathbf{2}} + (ENR-1)(\boldsymbol{F_{DUT}} - 1)\right]$$

$$\frac{\Delta G_{DUT}}{G_{DUT}} = \frac{\sqrt{2}\,\Delta_{Gain}}{ENR-1}\sqrt{ENR^2 + \boldsymbol{F_{DUT}}^{\mathbf{2}} + (ENR-1)(\boldsymbol{F_{DUT}} - 1)}$$

Errors due to gain inaccuracy of the measuring receiver that are common to all four power measurements and do not affect the determination of device gain. Any changes to the gain of the receiver as the four power measurements are being made, however, will affect the measurement.

Device Gain Sensitivity to Receiver Linearity Errors

Errors due to receiver linearity are not correlated since the power levels will differ. Linearity errors must be treated as independent.

$$\left(\frac{\partial G_{DUT}}{\partial \Delta_{lin}}\right)^2 = \left(\frac{G_{DUT}}{(ENR-1)kTB}\ kTB\right)^2 + \left(\frac{G_{DUT}}{(ENR-1)kTB}\ ENR\ kTB\right)^2 + \left(\frac{1}{(ENR-1)\mathrm{kTB}}\ (G_{DUT}kTB + \boldsymbol{N_{ADUT}})\right)^2 + \left(\frac{1}{(ENR-1)\mathrm{kTB}}\ (ENRG_{DUT}kTB + \boldsymbol{N_{ADUT}})\right)^2$$

A derivation similar to the one above for gain yields:

$$\frac{\Delta G_{DUT}}{G_{DUT}} = \frac{\sqrt{2}\,\Delta_{Lin}}{ENR-1}\sqrt{ENR^2 + \boldsymbol{F_{DUT}}^{\boldsymbol{2}} + (ENR-1)(\boldsymbol{F_{DUT}}-1)}$$

Sensitivity to mismatch errors

A derivation using the same methodology as that used for gain variations yields:

$$\Delta\boldsymbol{G_{DUT\ P1}} = \frac{\boldsymbol{\partial G_{DUT}}}{\boldsymbol{\partial \Delta_{\rho 1}}}\Delta_{\boldsymbol{\rho 1}} = \frac{G_{DUT}}{(ENR-1)kTB}\ kTB\ 2\rho_{Noff}\ \rho_R = \frac{\boldsymbol{G_{DUT}}}{\boldsymbol{(ENR-1)}}\ \ \boldsymbol{2\rho_{Noff}\ \rho_R}$$

$$\Delta\boldsymbol{G_{DUT\ P2}} = \frac{\boldsymbol{\partial G_{DUT}}}{\boldsymbol{\partial \Delta_{\rho 2}}}\Delta_{\boldsymbol{\rho 2}} = -\frac{G_{DUT}}{(ENR-1)kTB} ENR\ kTB\ 2\rho_{Non}\ \rho_R = -\frac{\boldsymbol{ENR\ G_{DUT}}}{\boldsymbol{(ENR-1)}}\ \ \boldsymbol{2\rho_{Non}\ \rho_R}$$

$$\Delta\boldsymbol{G_{DUT\ P3\ in}} = \frac{\boldsymbol{\partial G_{DUT}}}{\boldsymbol{\partial \Delta_{\rho 3}}}\Delta_{\rho 3\ in} = -\frac{1}{(ENR-1)\mathrm{kTB}}(G_{DUT}kTB + \boldsymbol{N_{ADUT}})\ 2\rho_{Noff}\ \rho_{DUTin}$$

$$\Delta\boldsymbol{G_{DUT\ P3\ in}} = \frac{\boldsymbol{\partial G_{DUT}}}{\boldsymbol{\partial \Delta_{\rho 3}}}\Delta_{\rho 3\ in} = -\frac{1}{(ENR-1)\mathrm{kTB}}(G_{DUT}kTB + (\boldsymbol{F_{DUT}} - \boldsymbol{1})\boldsymbol{G_{DUT}\ kTB})\ 2\rho_{Noff}\ \rho_{DUTin}$$

$$\Delta\boldsymbol{G_{DUT\ P3\ in}} = \frac{\boldsymbol{\partial G_{DUT}}}{\boldsymbol{\partial \Delta_{\rho 3}}}\Delta_{\rho 3\ in} = -\frac{G_{DUT}\boldsymbol{F_{DUT}}}{(ENR-1)}\ 2\rho_{Noff}\ \rho_{DUTin}$$

$$\Delta\boldsymbol{G_{DUT\ P3\ out}} = \frac{\partial G_{DUT}}{\partial \Delta_{\rho 3}}\Delta_{\rho 3\ out} = -\frac{G_{DUT}\boldsymbol{F_{DUT}}}{(ENR-1)}\ \ 2\ \rho_{DUTout}\ \rho_R$$

$$\Delta\boldsymbol{G_{DUT\ P4\ in}} = \frac{\partial G_{DUT}}{\partial \Delta_{\rho 4}}\Delta_{\rho 4\ in} = \frac{1}{(ENR-1)kTB}(\ ENRG_{DUT}kTB + \boldsymbol{N_{ADUT}})2\ \rho_{Non}\ \rho_{DUTin}$$

$$\Delta\boldsymbol{G_{DUT\ P4\ in}} = \frac{\partial G_{DUT}}{\partial \Delta_{\rho 4}}\Delta_{\rho 4\ in} = \frac{1}{(ENR-1)kTB}(\ ENRG_{DUT}kTB + (\boldsymbol{F_{DUT}} - \boldsymbol{1})\boldsymbol{G_{DUT}\ kTB})2\ \rho_{Non}\ \rho_{DUTin}$$

$$\Delta\boldsymbol{G_{DUT\ P4\ in}} = \frac{\partial G_{DUT}}{\partial \Delta_{\rho 4}}\Delta_{\rho 4\ in} = \frac{G_{DUT}(\ ENR + \boldsymbol{F_{DUT}} - \boldsymbol{1})}{ENR-1}2\rho_{Non}\ \rho_{DUTin}$$

$$\Delta\boldsymbol{G_{DUT\ P4\ in}} = \frac{\partial G_{DUT}}{\partial \Delta_{\rho 4}}\Delta_{\rho 4\ in} = G_{DUT}\left(1 + \frac{\boldsymbol{F_{DUT}}}{ENR-1}\right)2\rho_{Non}\ \rho_{DUTin}$$

$$\Delta\boldsymbol{G_{DUT\ P4\ out}} = \frac{\partial G_{DUT}}{\partial \Delta_{\rho 4}}\Delta_{\rho 4\ out} = G_{DUT}\left(1 + \frac{\boldsymbol{F_{DUT}}}{ENR-1}\right)2\ \rho_{DUTout}\ \rho_R$$

Case 1: The noise source reflection coefficient does not change between the hot and cold state

$$\Delta G_{DUT}{}^{2} = (\Delta \boldsymbol{G}_{\boldsymbol{DUT\ P1}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P2}})^{2} + (\Delta \boldsymbol{G}_{\boldsymbol{DUT\ P3\ in}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P4\ in}})^{2} + (\Delta \boldsymbol{G}_{\boldsymbol{DUT\ P3\ out}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P4\ out}})^{2}$$

In this case the mismatch error for P1 and P2 are the same, as are the mismatch errors for P3 and P4 are the same. There is partial cancellation of the effects as reflected by the above equation.

Case 2: The noise source reflection coefficient changes between its cold and hot state and the DUT has enough isolation for its output reflection coefficient to be independent from the input.

$$\Delta G_{DUT}{}^{2} = \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P1}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P2}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P3\ in}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P4\ in}}{}^{\boldsymbol{2}} + (\Delta \boldsymbol{G}_{\boldsymbol{DUT\ P3\ out}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P4\ out}})^{2}$$

There is still partial cancellation of the mismatch errors at the DUT output.

Case 3: The noise source reflection coefficient changes between its cold and hot state and the DUT has low solation so its output reflection coefficient depends on the input.

In this case, all power measurement errors due to mismatches are uncorrelated and

$$\Delta G_{DUT}{}^{2} = \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P1}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P2}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P3\ in}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P4\ in}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P3\ out}}{}^{\boldsymbol{2}} + \Delta \boldsymbol{G}_{\boldsymbol{DUT\ P4\ out}}{}^{\boldsymbol{2}}$$

Summary of Errors

Errors in Measuring Noise Factor

The noise factor of a device can be measured by taking four power measurements.

$$\boldsymbol{F_{DUT}} = \frac{(ENR_{Spec} - 1)(\boldsymbol{P_3} - \boldsymbol{P_1}) + \boldsymbol{P_2} - \boldsymbol{P_1}}{\boldsymbol{P_4} - \boldsymbol{P_3}}$$

		Normalized Errors in Measuring Noise Factor
		Error Component
Power subtraction uncertainty		$\frac{\Delta F_{DUT}}{F_{DUT}} = \frac{M}{F_{DUT}}\left[\frac{ENR - F_{DUT}}{(ENR-1)G_{DUT} + M} - F_{DUT}\frac{(ENR-1)M}{[(ENR-1)G_{DUT} + M]^2}\right]$
ENR inaccuracy		$\frac{\Delta F_{DUT}}{F_{DUT}} = \frac{ENR}{(ENR-1)}\frac{G_{DUT}-1}{G_{DUT}}\Delta_{ENR}$
Receiver Gain Inaccuracy		$\frac{\Delta F_{DUT}}{F_{DUT}} = \frac{\sqrt{2}\,\Delta_{Gain}}{(ENR-1)}\sqrt{\left(\frac{ENR}{G_{DUT}\,F_{DUT}}\right)^2 + (ENR - 1 + F_{DUT})^2}$
Receiver Linearity		$\frac{\Delta F_{DUT}}{F_{DUT}} = \frac{\sqrt{2}\,\Delta_{Lin}}{(ENR-1)}\sqrt{\left(\frac{ENR}{G_{DUT}\,F_{DUT}}\right)^2 + (ENR - 1 + F_{DUT})^2}$
Mismatch between the noise source and the receiver	**Noise Source Off**	$\frac{\Delta F_{DUT}}{F_{DUT}} = -\frac{ENR}{(ENR-1)}\frac{2\rho_{Noff}\,\rho_R}{G_{DUT}\,F_{DUT}}$
	Noise source On	$\frac{\Delta F_{DUT}}{F_{DUT}} = \frac{ENR}{(ENR-1)}\frac{2\rho_{Non}\,\rho_R}{G_{DUT}\,F_{DUT}}$
Mismatch between the noise source and the DUT input	**Noise Source Off**	$\frac{\Delta F_{DUT}}{F_{DUT}} = \left(1 + \frac{F_{DUT}}{(ENR-1)}\right) 2\,\rho_{Noff}\,\rho_{DUTin}$
	Noise source On	$\frac{\Delta F_{DUT}}{F_{DUT}} = -\left[\frac{ENR + 1 - F_{DUT}}{(ENR-1)}\right] 2\,\rho_{Non}\,\rho_{DUTin}$
Mismatch between the DUT output and the receiver	**Noise Source Off**	$\frac{\Delta F_{DUT}}{F_{DUT}} = \left(1 + \frac{F_{DUT}}{(ENR-1)}\right) 2\,\rho_{DUTout}\,\rho_R$
	Noise source On	$\frac{\Delta F_{DUT}}{F_{DUT}} = -\left[\frac{ENR + 1 - F_{DUT}}{(ENR-1)}\right] 2\,\rho_{DUTout}\,\rho_R$

Table 5-2. Errors in measuring DUT noise factor.

Errors in Measuring Device Gain

The gain a device is computed from four power measurements.

$$\boldsymbol{G_{DUT}} = \frac{\boldsymbol{P_4 - P_3}}{\boldsymbol{P_2 - P_1}}$$

		Normalized Errors in Measuring Gain
		Error Component
Power Subtraction uncertainty (SUB DANL Floor)		$\frac{\Delta G_{DUT}}{G_{DUT}} = -\frac{(G_{DUT}-1)M}{G_{DUT}} \frac{(\mathrm{ENR}-1)}{[(\mathrm{ENR}-1)+M]^2}$
ENR inaccuracy		$\frac{\Delta G_{DUT}}{G_{DUT}} = 0$
Receiver Gain Inaccuracy		$\frac{\Delta G_{DUT}}{G_{DUT}} = \frac{\sqrt{2}\,\Delta_{Gain}}{ENR-1}\sqrt{ENR^2 + {F_{DUT}}^2 + (ENR-1)(F_{DUT}-1)}$
Receiver Linearity		$\frac{\Delta G_{DUT}}{G_{DUT}} = \frac{\sqrt{2}\,\Delta_{Lin}}{ENR-1}\sqrt{ENR^2 + {F_{DUT}}^2 + (ENR-1)(F_{DUT}-1)}$
Mismatch between the noise source and the receiver	Noise Source Off	$\frac{\Delta G_{DUT}}{G_{DUT}} = \frac{1}{(ENR-1)}\ 2\rho_{Noff}\,\rho_R$
	Noise source On	$\frac{\Delta G_{DUT}}{G_{DUT}} = -\frac{ENR}{(ENR-1)}\ 2\rho_{Non}\,\rho_R$
Mismatch between the noise source and the DUT input	Noise Source Off	$\frac{\Delta G_{DUT}}{G_{DUT}} = -\frac{F_{DUT}}{(ENR-1)}\ 2\,\rho_{Noff}\,\rho_{DUTin}$
	Noise source On	$\frac{\Delta G_{DUT}}{G_{DUT}} = \left(1+\frac{F_{DUT}}{ENR-1}\right) 2\,\rho_{Non}\,\rho_{DUTin}$
Mismatch between the DUT output and the receiver	Noise Source Off	$\frac{\Delta G_{DUT}}{G_{DUT}} = -\frac{F_{DUT}}{(ENR-1)}\ 2\,\rho_{DUTout\,off}\,\rho_R$
	Noise source On	$\frac{\Delta G_{DUT}}{G_{DUT}} = \left(1+\frac{F_{DUT}}{ENR-1}\right) 2\,\rho_{DUTout\,on}\,\rho_R$

Table 5-3. Errors in measuring DUT gain.

Statistical Distribution of Measurement Errors

The statistical distribution of measurement is a complex topic, a treatment of which is beyond the scope of this paper. A summary of the error distributions encountered in the measurement of noise fugure and gain is included for clarity and completeness.

ENR

The error in determine the ENR of a noise source is the difference between the ENR specified by the noise source manufacturer and the actual value. This error has many influencing factors, including measurement frequency temperature, current, noise source aging, etc. Manufacturers typically specify noise sources have excess noise ratio within ± 1dB from a specified value. A good assumption is that the ± 1dB spread covers 95% of units with uniform distributions (σ=0.5 dB).

Receiver Gain Error

This is the error with which a receiver can measure a power level coming from a prefect Z_0 (Usually 50Ω) Ohm source. It is typically includes a reference error and a frequency response error. This error can also be further improved by making a local calibration using a power meter and a signal source. Receiver gain errors can be assumed to be normally distributed. If a 95 percentile number is given, it can be assumed to be equal to 2 σ.

Receiver Linearity

Linearity was formerly a big error, especially in spectrum analyzers that used analog circuits for their log amplifiers. Today's receivers use D-A converters and DSP. The linearity is much better but can still be significant. Sources of linearity errors include digitizer quantization errors, intermodulation and spurious.

Receiver linearity at high input levels is usually limited by its spurious-free dynamic range (SFDR). Noise figure measurements, however, often measure power levels that are close to the instrument's noise floor (One of the four measurements, P_1 is indeed a measure of the instrument's noise floor). Linearity takes on a different character at these low levels.

1. Linearity affects a measuring instrument's ability to accurately determine the difference between two power levels. It is often lumped into the instrument uncertainty for gain and for noise figure measurements.
2. While it is customary to examine the linearity of electrical devices in terms of their 2nd, 3rd, 4th … order intercept points, devices with Analog to digital converters have some low-level quantization errors limit linearity at the low levels that are used in noise figure measurements.
3. Tektronix Spectrum analyzers use a variety of techniques to drop the quantization errors to levels below the instrument's noise floor. A good upper bound for the linearity errors:
    a. Expressed as a power ratio: $\Delta_{Lin} = \frac{F_R}{G_{DUT}F_{DUT}}$
    b. Expressed in dB: $\Delta_{LindB} = F_{RdB} - G_{DUTdB} - F_{DUTdB}$

The linearity errors can be assumed to be normally distributed.

Mismatch Errors

Mismatch errors are of the form

$$\Delta_\rho = 1 - \frac{1}{(1+\rho_1\rho_2)^2} = -2\ \rho_1\rho_2$$

ρ_1 and ρ_2 are vector quantities that have a normally distributed magnitude and an phase that is uniformly distributed over 0 to 2π radians.

It can be shown that this type of error has distribution that is U-shaped as is shown in Figure 5-1. It can also be shown that this kind of distribution has a standard deviation of $\sigma = \sqrt{2}\rho_1\rho_2$, and that 95% of all occurrences will lie within $2\sqrt{2}\rho_1\rho_2$ of the mean.

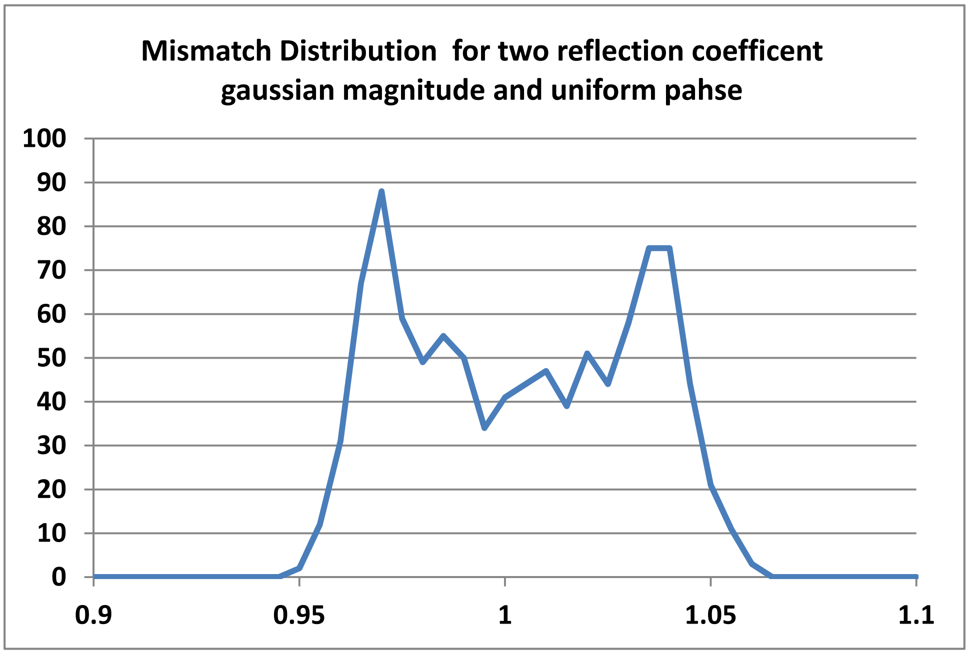

Figure 5-1. Histogram of mismatch for two reflecting coefficients with Gaussian magnitude and uniform phase (ρ_{mean}=.2, σ_ρ=0.02).

Coverage Factor

In general, the value of the coverage factor k is chosen on the basis of the desired level of confidence to be associated with the interval defined by U = ku_c. Typically, k is in the range 2 to 3. When the normal distribution applies and uc is a reliable estimate of the standard deviation of y, U = 2 u_c (i.e., k = 2) defines an interval having a level of confidence of approximately 95 %, and U = 3 u_c (i.e., k = 3) defines an interval having a level of confidence greater than 99 %.

A common approach for errors with distributions that are not normal is to use a coverage factor that allows a similar level of confidence.

Distribution	Standard Deviation	
Normal	σ	Noise
Constrained distributions: -a<x<a		
Rectangular (Uniform)	$\frac{a}{\sqrt{3}}$	Frequency counter LSB
Triangular	$\frac{a}{\sqrt{6}}$	Results from the combination of two uniform distributions
Cosine	$\frac{a}{\sqrt{3}}\sqrt{1-\frac{6}{\pi^2}}$	Distribution of X-values that result from uniformly distributed phase
U-shaped	$\frac{a}{\sqrt{2}}$	Probability of the values of sinusoids

Table 5-4. Some typical error distributions

Example of Contributions to Noise Figure Measurement

The following is an illustrative example of the sources of error in a noise figure measurement. The table details parameters for the DUT, noise source and the measuring receiver.

Device Under Test	DUT NF	1	dB
	DUT Gain	20	dB
	DUT Input Match	1.5	VSWR
	DUT Output Match	1.5	VSWR
Noise Source	ENR	15	dB
	ENR Uncertainty	0.15	dB
	Noise Source match	1.15	VSWR
Measuring Instrument	Noise Figure	10	dB
	Instrument Uncertainty for NF	.02	dB
	Instrument Uncertainty for Gain	.07	dB
	SUB DANL Floor	13	dB
	Input Match	1.45	VSWR

The uncertainty in the noise source ENR and the mismatch errors are the largest contributor to uncertainty in noise figure. Mismatch errors are the largest contributors to gain uncertainty.

Contributions to uncertainty in NF and gain		
Error Contributions	**NF Error (dB)**	**Gain Error (dB)**
ENR	0.153	0.000
Gain	0.030	0.030
Linearity	0.105	0.104
Noise subt. Uncertainty	0.003	0.018
Mismatches	0.030	0.454
Total	**0.189**	**0.465**