PASSIVE AND ACTIVE FILTERS LAB

BEFORE YOU BEGIN

PREREQUISITE LABS
► Advanced MATLAB
► Introduction to Oscilloscope
► Introduction to Arbitrary/Function Generator

EXPECTED KNOWLEDGE
► Laplace transform for circuit analysis
► Transfer functions
► Filtering concepts

EQUIPMENT
► Digital Multimeter
► Programmable Power Supply
► AFG3000 Series Arbitrary/Function Generator
► TDS3034B Digital Phosphor Oscilloscope

MATERIALS
► Complete set of resistors
► Complete set of capacitors
► Set of Operational Amplifiers
► BNC to Alligator cables
► Alligator to Alligator cables
► Female Mini Plug
► Oscilloscope Probes
► 3 ½ floppy (optional)
► Speaker Control Box
► Sound Card
► Audio Cable
► Solderless Breadboard

OBJECTIVES
After completing this lab you should:
► Be familiar with the characteristics of basic filter circuits.
► Know how to construct basic filter circuits for a given application.
► Know how to analyze the properties of an analog filter using transfer function analysis.

INTRODUCTION
Passive and active filters play a vital role in many aspects of electrical engineering and are used in many applications including telecommunications and signal processing. Filters are used to
separate one or more frequency components of a signal from that of other signals. Operational amplifiers play a key role in analog filter design and give engineers a great deal of flexibility when designing analog filters.

In this lab you will be working with basic filter circuits that are commonly used in applications. You should become very familiar with these circuits and learn the fundamentals of circuit analysis using transfer functions and Bode plots.

**PRELAB**

**MATLAB**

*The BODE Command*

MATLAB’s Signal Control Box has many tools to assist in the design and analysis of filters. One of the most useful tools is the BODE command. This lab will only cover a portion of the many different ways in which this command can be used.

The BODE command is used to create Bode plots in MATLAB. The basic syntax for the command is:

```
[mag, phase, w] = bode(num, den);
```

where

- **mag** is the magnitude of the transfer function
- **phase** is the phase shift in degrees of the transfer function
- **w** is a row vector containing the frequency values in radians per second
- **num** is a row vector containing the polynomial coefficients of the numerator of the transfer function
- **den** is a row vector containing the polynomial coefficients of the denominator of the transfer function

It is important to notice that **mag** is the magnitude of the transfer function. It is not the absolute value of the magnitude in decibels. To obtain the absolute value of the amplitude in decibels, \( |H(s)|_{dB} \), you must perform the mathematical conversion. For example, you could type

```
magdb = 20 * log10(mag);
```

at the MATLAB command prompt.

The BODE command can also be used with the following syntax:

```
[mag, phase] = bode(num, den, w);
```

When used as an argument, **w** can be used to specify a range of frequencies, in radians per second, such as

```
[mag, phase] = bode(num, den, [10, 1e6]);
```
which defines a frequency range from 10 rad/s to 1 Mrad/s. The variable w can also be a row vector of frequency values to use, such as

\[
w = \text{logspace}(1, 6, 25);
\]

\[
[\text{mag}, \text{phase}] = \text{bode}(\text{num, den, w});
\]

**The LOGSPACE Command**

The LOGSPACE command is used to create a vector of logarithmically equally spaced points. The syntax for the LOGSPACE command is

\[
\text{logspace}(d1, d2, N)
\]

where

- \(d1\) is the starting exponent value
- \(d2\) is the ending exponent value
- \(N\) is the number of points generated

If \(N\) is omitted, LOGSPACE generates 50 points.

For example, if you wanted to create 50 logarithmically equally spaced frequencies between 10 rad/s and 1 krad/s, you would use the command

\[
w = \text{logspace}(1, 3);
\]

since \(\log_{10} 10 = 1\) and \(\log_{10} 1000 = 3\).

If you wanted to create a vector of 30 logarithmically equally spaced frequencies between 50 rad/s and 5 krad/s, you could use the command

\[
w = \text{logspace}(\log_{10}(50), \log_{10}(5e3), 30);
\]

As an example, the following script shows you how to create a magnitude and a phase Bode plot for the transfer function \(H(s) = \frac{s}{s + 1000}\), using 30 points from 5 rad/s to 10 krad/s:

\[
\begin{align*}
\text{num} &= [1 0]; \quad \% \text{Numerator: } 1 \, s^1 + 0 \, s^0 \\
\text{den} &= [1 1000]; \quad \% \text{Denominator: } 1 \, s^1 + 1000 \, s^0 \\
w &= \text{logspace}(\log_{10}(5), 4, 30); \% 10^4 = 10 \, k \\
[\text{mag}, \text{phase}] &= \text{bode}(\text{num, den, w}); \\
\text{magdb} &= 20 \times \log_{10}(\text{mag}); \\
\text{subplot}(2,1,1); \\
\text{semilogx}(w, \text{magdb}); \\
\text{grid}; \\
\text{axis}([5 \, 10000 \, -50 \, 0]); \\
\text{title}(\text{'Bode Plots for } H(s) = s / s + 1000') \\
\text{xlabel}(\text{'Frequency, rad/s'}); \\
\text{ylabel}(\text{'Magnitude, dB'});
\end{align*}
\]
subplot(2,1,2);
semilogx(w, phase);
grid;
axis([5 10000 0 100]);
xlabel('Frequency, rad/s');
ylabel('Phase, deg');

The Bode plots produced by this script file are shown in Figure 1.

![Bode Plots for H(s) = s / s + 1000](image1)

**Figure 1. Example Bode Plots**

**Answer Questions 1 – 2.**

**Using LabVIEW to Create Bode Plots**

In this lab you will be using a LabVIEW program to create Bode plots for the circuits you build in the PRELAB exercise. To do so, run LabVIEW and open BODEPLOT.VI in the directory >Program Files>National Instruments>LabVIEW 6>user.lib. The Bode plot VI should look like the one in Figure 2. The Bode plot VI uses the AFG3000 Series Arbitrary/Function Generator to sweep through a range of frequencies and measures the voltage on channels 1 and 2 of the TDS3034B Oscilloscope.
If you look to the right side of the Bode plot VI, you will see four text boxes. These are used to enter the starting frequency, the stopping frequency, the number of points to capture and the input voltage.

Just below the text boxes you will see an option to save the data. If this option is checked, the VI will prompt you for a filename after the data has been collected. The data is saved into a comma-delimited file as coordinate ordered pairs. You would use this option if you wanted to import the data for the Bode plot into MATLAB or Excel.

Below the text boxes you will see three command buttons. Clicking the START button activates the VI. Once the VI starts, there is no way to stop the process; the process will continue until the number of data points specified has been collected. The PRINT button sends the information displayed in the VI to the printer. After the VI is finished running, clicking on the EXIT button closes the VI.

When connecting circuits to be used with the Bode plot VI, always use oscilloscope probes and set the probes to X10 attenuation. Channel 1 of the oscilloscope must be used to measure the output voltage and channel 2 must be used to measure the input voltage.
On the right side of the Bode plot VI are two graphs. The top graph displays the signals that are present on channels 1 and 2 of the oscilloscope. As the VI runs you will be able to see how the different frequencies affect the output voltage.

The lower graph is the Bode plot of the circuit under test. This graph is not updated until the VI is finished gathering all the data points.

The Bode plot VI displays the frequency in Hertz. If you design your circuit with a cutoff frequency of 500 rad/s, the Bode plot will show a cutoff frequency of 80 Hz at –3 dB.

**Answer Question 3.**

**RC Low Pass Filters**

The circuit in Figure 3 is a series RC low pass filter. When the frequency of $V_S$ is 0 rad/s (DC), the capacitor has infinite impedance and acts like an open circuit and $V_O = V_S$. At high frequencies, the capacitor has very little impedance and acts like a short circuit. Therefore, as the frequency of $V_S$ approaches $\infty$, the potential of $V_O$ approaches 0 V.

The cutoff frequency, $\omega_c$, of an RC low pass filter is given by $\omega_c = \frac{1}{RC}$.

**Answer Questions 4 – 5.**

![Figure 3. A Series RC Low Pass Filter.](image)

Build an RC low pass filter using the values obtained from Question 5.

**RC High Pass Filters**

The circuit in Figure 4 is an RC high pass filter. When the frequency of $V_S$ is 0 rad/s, the capacitor acts like an open circuit and the potential at $V_O$ is equal to 0 V. As the frequency of $V_S$ approaches $\infty$, the impedance of the capacitor decreases and $V_O$ approaches $V_S$.

The equation for the cutoff frequency, $\omega_c$, of an RC high pass filter is the same as for an RC low pass filter, $\omega_c = \frac{1}{RC}$.
Answer Questions 6 - 8.

Build an RC high pass filter using the values obtained from Question 7.

**RC Differentiators**

The circuit in Figure 5 is an RC differentiator. Performing nodal analysis at the inverting node yields the equation:

\[ C \frac{\partial V_s}{\partial t} + \frac{V_o}{R} = 0. \]

Solving for \( V_o \):

\[ \frac{V_o}{R} = -C \frac{\partial V_s}{\partial t} \]

\[ V_o = -RC \frac{\partial V_s}{\partial t} \]

Note that the output voltage is equal to the derivative of the input voltage.

Since this filter does not have a theoretical maximum value, we cannot define a cutoff frequency. Instead, we define a unity-gain frequency, \( \omega_o \), which is defined as the frequency at which the gain is unity. It can be shown that this frequency is given by \( \omega_o = \frac{1}{RC} \). When the impedance of the capacitor is equal to the impedance of the resistor (\( Z_C = Z_R \)), the circuit provides a gain of 1, or 0 dB, and is said to have unity gain.
When the impedance of the capacitor is less than the impedance of the resistor \((Z_C < Z_R)\), the circuit amplifies. When the impedance of the capacitor is greater than the impedance of the resistor \((Z_C > Z_R)\), the circuit attenuates.

**Answer Questions 9 - 10.**

Build an RC differentiator using the values obtained from Question 10.

**RC INTEGRATORS**

The circuit shown in Figure 6 is an RC integrator. Performing nodal analysis at the inverting node yields the equation:

\[
\frac{V_S}{R} + C \frac{\partial V_o}{\partial t} = 0
\]

\[
C \frac{\partial V_o}{\partial t} = -\frac{V_S}{R}
\]

\[
\frac{\partial V_o}{\partial t} = -\frac{V_S}{RC}
\]

\[
V_o = -\frac{1}{RC} \int V_S \, dt
\]

The output voltage is equal to the integral of the input voltage.

**Answer Questions 11 - 12.**

![Figure 6. An RC Integrator.](image)

Build an RC integrator using the values obtained from Question 12.

**ACTIVE LOW PASS FILTERS**

By placing a resistor in parallel with the capacitor of an RC integrating circuit, we can create an active (inverting) low pass filter, as shown in Figure 7. An active filter is a filter that contains an active circuit element that is usually an operational amplifier. The gain of active filters may be larger or smaller than unity.
The cutoff frequency of the filter shown in Figure 7 is \( \omega_c = \frac{1}{R_2 C} \), and the DC gain of the circuit is \( A = -\frac{R_2}{R_1} \).

**Answer Questions 13 - 14.**

![Figure 7. An Active Low Pass Filter.](image)

Build an active low pass filter using the values obtained from Question 14.

**ACTIVE HIGH PASS FILTERS**

By placing a resistor in series with the capacitor in an RC differentiator, we can create an active high pass filter. Figure 8 shows a circuit diagram of an active high pass filter. The cutoff frequency of the filter is \( \omega_c = \frac{1}{R_1 C} \), and the DC gain of the filter is \( A = -\frac{R_2}{R_1} \).

**Answer Questions 15 – 16.**

![Figure 8. An Active High Pass Filter.](image)

Build an active high pass filter using the values obtained from Question 16.
ACTIVE BAND PASS FILTERS

A band pass filter only allows a band of frequencies to pass through. All other frequencies are filtered out. Figure 9 shows an example of an active band pass filter.

![Figure 9. An Active Band Pass Filter.](image)

The band pass filter is a combination of the low and high pass filters. When designing and working with band pass filters, there are six attributes that specify the key aspects of the filter performance: the lower cutoff frequency, the upper cutoff frequency, the center frequency, the passband gain, the bandwidth, and the quality factor.

If you treat the band pass filter as an inverting op amp, the impedance $Z_1$ consists of $R_1$ in series with $C_1$. This portion of the circuit determines the lower cutoff frequency $\omega_{cl} = \frac{1}{R_1C_1}$. The impedance $Z_2$ consists of $R_2$ in parallel with $C_2$. This portion of the circuit determines the upper cutoff frequency $\omega_{ch} = \frac{1}{R_2C_2}$.

The center frequency $\omega_o$ is the resonant frequency of the filter. For example, a 5 k rad/s band pass filter has a center frequency of 5 k rad/s. The center frequency is the geometric mean of the cutoff frequencies $\omega_o = \sqrt{\omega_{cl}\omega_{ch}}$.

The bandwidth $\beta$ of the filter is given by $\beta = \omega_{ch} - \omega_{cl}$.

The quality factor $Q$ is the ratio of the center frequency $\omega_o$ to the bandwidth $\beta$, $Q = \frac{\omega_o}{\beta}$.

Answer Questions 17 - 19.

Build an active band pass filter using the values obtained from Questions 18 and 19.
**ACTIVE TONE CONTROL**

One application of active filters is tone control. The most common type of tone control is bass and treble control. Figure 10 shows a schematic of a bass and treble controller. The circuit in Figure 10 allows independent adjustment of gain of the lower frequency range (bass) and higher frequency range (treble) of the signal.

![Bass and Treble Control](image)

**Bass Control**

At the low end of the audio signal, the capacitors act as open circuits. As a result, the feedback from the op amp is characterized by $R_1$ and $R_2$ (see Figure 11). At low frequencies, the op amp acts like an inverting amplifier. The magnitude of the gain for low frequencies is $\frac{R_1}{R_1 + R_2} \leq |A_2| \leq \frac{R_1 + R_2}{R_1}$. The lower limit is referred to as maximum cut, and the upper limit is referred to as maximum boost.

![Bass Control](image)
Figure 11. Bass Controler at Low Frequencies.

For example, if \( R_1 = 11 \text{ k}\Omega \) and \( R_2 = 100 \text{ k}\Omega \), then the maximum cut would be

\[
20 \log_{10} \left( \frac{11,000}{11,000 + 100,000} \right) \approx -20 \text{ dB},
\]

and the maximum boost would be

\[
20 \log_{10} \left( \frac{11,000 + 100,000}{11,000} \right) \approx 20 \text{ dB}.
\]

As the frequency increases, \( R_2 \) is eventually short-circuited by \( C_1 \). The frequency \( \omega_B \) at which the lower end of the audio signal is effectively amplified is \( \omega_B = \frac{1}{R_2 C_1} \). At frequencies above \( \omega_B \) the bode plot tends to flat line with unity gain.

Answer Questions 20 – 22.

Treble Control

At the higher end of the audio signal the capacitors act like short circuits as shown in Figure 12. If \( R_4 \ll R_1 + R_3 + 2R_5 \), then the gain for the treble controller is within the range of

\[
\frac{R_3}{R_1 + R_3 + 2R_5} \leq A_T \leq \frac{R_1 + R_3 + 2R_5}{R_3}.
\]

The frequency \( \omega_T \) at which the higher end of the audio signal is effectively amplified is \( \omega_T = \frac{1}{R_3 C_2} \).

Answer Questions 23 – 25.

![Figure 12. Treble Controller at High Frequencies.](image)

Build an active bass and treble control using the values obtained from Questions 20 – 25.
LAB

RC LOW PASS FILTERS

Answer Question 26.

Use the Bode plot VI to create a Bode plot of the RC low pass filter you built for the PRELAB exercise (Figure 3) with a frequency range of $5 \text{ Hz} \leq \frac{\omega}{2\pi} \leq 10 \text{ kHz}$ at 25 equally spaced points and an amplitude of 2 V. Save the data generated by the Bode plot VI in a file to be imported into MATLAB.

Answer Questions 27 – 29.

RC HIGH PASS FILTERS

Answer Question 30.
Use the Bode plot VI to create a Bode plot of the RC high pass filter (Figure 4) with a frequency range of $5 \text{ Hz} \leq \frac{\omega}{2\pi} \leq 10 \text{ kHz}$ at 25 equally spaced points and amplitude of 2 V. Save the data generated by the Bode plot VI in a file to be imported into MATLAB.

**Answer Questions 31 – 33.**

**RC DIFFERENTIATORS**

**Answer Question 34.**

Use the Bode plot VI to create a Bode plot of the RC differentiator (Figure 5) with a frequency range of $5 \text{ Hz} \leq \frac{\omega}{2\pi} \leq 3 \text{ kHz}$ at 25 equally spaced points and amplitude of 2 V. Save the data generated by the Bode plot VI in a file to be imported into MATLAB.

**Answer Questions 35 – 36.**

**RC INTEGRATORS**

**Answer Question 37.**

Use the Bode plot VI to create a Bode plot of the RC integrator (Figure 6) with a frequency range of $100 \text{ Hz} \leq \frac{\omega}{2\pi} \leq 10 \text{ kHz}$ at 25 equally spaced points and amplitude of 10 V. Save the data generated by the Bode plot VI in a file to be imported into MATLAB.

**Answer Questions 38 – 40.**

**ACTIVE LOW PASS FILTERS**

**Answer Question 41.**

Use the Bode plot VI to create a Bode plot of the active low pass filter (Figure 7) with a frequency range of $5 \text{ Hz} \leq \frac{\omega}{2\pi} \leq 10 \text{ kHz}$ at 25 equally spaced points and amplitude of 2 V. Save the data generated by the Bode plot VI in a file to be imported into MATLAB.

**Answer Questions 42 – 44.**

**ACTIVE HIGH PASS FILTERS**

**Answer Question 45.**

Use the Bode plot VI to create a Bode plot of the active high pass filter (Figure 8) with a frequency range of $5 \text{ Hz} \leq \frac{\omega}{2\pi} \leq 10 \text{ kHz}$ at 25 equally spaced points and amplitude of 2 V. Save the data generated by the Bode plot VI in a file to be imported into MATLAB.

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Answer Questions 46 – 48.

**ACTIVE BAND PASS FILTERS**

Answer Question 49.

Use the Bode plot VI to create a Bode plot of the active band pass filter (Figure 9) with a frequency range of $5 \text{ Hz} \leq \frac{\omega}{2\pi} \leq 10 \text{ kHz}$ at 25 equally spaced points and amplitude of 2 V. Save the data generated by the Bode plot VI in a file to be imported into MATLAB.

Answer Questions 50 – 54.

**ACTIVE TONE CONTROL**

*Bass Control*

Center the treble potentiometer and turn the bass potentiometer all the way counter clockwise. Use the Bode plot VI to create a Bode plot of the bass control with a frequency range of $10 \text{ Hz} \leq \frac{\omega}{2\pi} \leq 10 \text{ kHz}$ at 25 equally spaced points and amplitude of 2 V. Save the data generated by the Bode plot VI in a file to be imported into MATLAB. Turn the bass potentiometer so that it is half way between center and the most counter clockwise position. Create another Bode plot and save the data. Repeat this procedure with the bass potentiometer centered, half way between center and the most clockwise position, and in the most clockwise position.

*Treble Control*

Center the bass potentiometer and turn the treble potentiometer all the way counter clockwise. Use the Bode plot VI to create a Bode plot of the treble control with a frequency range of $10 \text{ Hz} \leq \frac{\omega}{2\pi} \leq 10 \text{ kHz}$ at 25 equally spaced points and amplitude of 2 V. Save the data generated by the Bode plot VI in a file to be imported into MATLAB. Turn the treble potentiometer so that it is half way between center and the most counter clockwise position. Create another Bode plot and save the data. Repeat this procedure with the treble potentiometer centered, half way between center and the most clockwise position, and in the most clockwise position.

Answer Question 55.
TESTING THE ACTIVE TONE CONTROLLER

Next, you will connect the active tone controller, as shown in Figure 15, to an audio signal and listen to the output while adjusting the bass and treble controllers.

Make sure the switch on the speaker control box is in the proper position to connect the sound card to the speakers. Start the Windows Media Player by clicking on Start → Programs → Accessories → Entertainment → Windows Media Player. After Windows Media Player loads, click on the Radio button. This will start IE and load http://windowsmedia.com/radiotuner/default.asp, where you can select a radio station to listen to. Select a radio station from the list on the left and click on Play at the bottom of the screen. After a few seconds, you should be able to hear the radio broadcast.

Since we only have one tone controller, we can only adjust one channel. Therefore, only the left channel of the audio signal will be connected to your circuit. Position the switch on the speaker control box to disconnect the sound card from the speakers. Use a BNC to alligator cable to connect the speaker control box to the tone controller. This will serve as the input source to the circuit, $V_S$. Connect the BNC end of the cable to terminal L-OUT on the speaker control box. Connect the positive lead as if it was the positive lead of $V_S$ and the negative lead to ground.
Use another BNC to alligator cable to connect the tone controller to the speakers. Connect the BNC of the cable to terminal L-IN of the speaker control box. Connect the positive lead to $V_O$ and the negative lead to ground.

You should now be able to hear the radio broadcast over the left speaker at this point. Connect channel 1 of the oscilloscope to $V_O$ and channel 2 to $V_S$. Adjust the oscilloscope so that both waves are clearly visible, as in Figure 16. Make sure that both channels are set to the same scale factor, such as 50.0 mV, so you can see the difference in amplitude of the two signals. Start with the time scale at 10.0 ms. Adjust the time scale until the oscilloscope displays a good representation of the signal. The time scale value may vary depending on the signal you are controlling. As you adjust both potentiometers you should be able to hear and see how it affects the output signal.

Answer Questions 56 – 57.

![Figure 15. Input and Output Connectinos to Tone Controller.](image-url)
Figure 16. Input and Output of Active Tone Controller.